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MIDDLE EAST UNIVERSITY
Amman - Jordan

An Analysis of Scientific Articles on Mathematics Misconception: A Bibliometric Research

تحليل الأبحاث العلمية حول المفاهيم الخاطئة في الرياضيات: بحث ببليومتري

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A Thesis Submitted in Partial Fulfillment of the Requirements of the Master's Degree in Curriculum and Teaching Methods

Department of Administration and Curriculum
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Authorization

I, **Rana Jubraeil Yacoub Aleifat**, authorize the Middle East University to provide copies of my thesis on paper and electronically to libraries, organizations, and institutions concerned with scientific study and studies upon request.

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This thesis: "**An Analysis of Scientific Articles on Mathematics Misconception: A Bibliometric Research**" was successfully defended and approved in 27 May 2024.

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Dedication

To my father.

To my mother.

To my husband.

To my daughters.

To my brothers and sisters,

To all the loved ones…

To myself…

I dedicate this thesis.

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An Analysis of Scientific Articles on Mathematics Misconception: A Bibliometric Research

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Abstract

The purpose of this study is to conduct a bibliometric analysis of the research published in the field of mathematics misconception in the period 1947 to 2023, to identify the general knowledge structure and participation in research publication. An analytical approach was used based on Scopus database data. This study used mixed methods; quantitative method to summarize the articles using bibliometric analysis, and qualitative method to analyse the content of the most cited papers on mathematics misconception. An analysis of the data was conducted by analyzing the year of publication, the type of publication, the title of the research product, the institution and country where the researcher belongs, the number of citations, and the collaborative interactions between authors and their affiliated institutions. Additionally, the most frequently used keywords were analysed to identify the most common mathematics misconception. There were 525 research papers related to mathematics misconception, of which 447 were published in classified journals and 78 in conference proceedings indexed in Scopus. Research publications related to mathematics misconception have clearly increased over time, as one paper was published in 1947-1962 and 377 papers were published in 2011-2023. Additionally, most researchers and educational institutions publishing papers about mathematics misconceptions were from the USA, England, and Turkey. The most cited papers were those belonging to American researchers. Among the most common subtopics related to mathematics misconceptions were common misconception, reconstructing understanding of mathematical concepts, and learning and teaching about functions and graphs. Students, education, and teaching were the most commonly used keywords. The qualitative analysis of the ten most important research papers according to the number of citations showed the presence of (23) common mathematics misconceptions, classified into four categories: general mathematics misconception, algebraic mathematics misconception, trigonometric mathematics misconception, and calculus mathematics misconception. This study recommends that researchers should extend their investigations of bibliometric analysis to other fields of education given the valuable insights gained from the bibliometric analysis conducted in mathematics education.

Keywords: Mathematics Misconception, Bibliometric Analysis, Scientific Articles.

تحليل الأبحاث العلمية حول المفاهيم الخاطئة في الرياضيات: بحث ببليومتري

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إشراف: الدكتور أحمد عبد السميع طبية

الملخّص

تهدف هذه الدراسة إلى إجراء تحليل ببليومتري للنتاج البحثي المنشور في مجال الأخطاء المفاهيمية الرياضية في الفترة 1947 وحتى 2023، وذلك للتعرف على البناء المعرفي العام، والتشاركية في النشر البحثي. وقد اتبعت الدراسة المنهج التحليلي القائم على البيانات المتحصلة من قواعد بيانات Scopus. في هذه الدراسة تم استخدام التحليل المزجي؛ التحليل الكمي لتلخيص الأبحاث العلمية حول المفاهيم الخاطئة في الرياضيات باستخدام التحليل الببليومتري للأبحاث، والتحليل النوعي لتحليل محتوى أهم الأبحاث التي تناولت موضوع المفاهيم الخاطئة في الرياضيات. تم تحليل تلك البيانات وفقًا لسنة النشر، نوع النشر، عنوان النتاج البحثي، المؤسسة والبلد التابع لها الباحث، عدد الاستشهادات، والتشابكات التعاونية المختلفة بين المؤلفين والمؤسسات البحثية والتعليمية التابعين لها. إضافة إلى ذلك فقد تم تحليل الكلمات المفتاحية الأكثر استخدامًا لتحديد أكثر الموضوعات انتشارًا والمرتبطة بالأخطاء المفاهيمية الرياضية. وقد بلغ النتاج البحثي الكلي المرتبط بالأخطاء المفاهيمية الرياضية (525)، كان منه (447) ورقة بحثية منشورة بمجلات مصنفة و (78) ورقة منشورة بمؤتمرات مفهرسة بقواعد بيانات Scopus. وقد لوحظ تزايد واضح في النشر البحثي المرتبط بالأخطاء المفاهيمية الرياضية، حيث بدأ بورقة واحدة بالفترة (1962 – 1947)، وانتهى بعدد من الأوراق بلغ (377) ورقة منشورة بالفترة (2023–2011). كما أظهرت النتائج أن أكثر الباحثين والمؤسسات التعليمية نشرًا للأوراق المتعلقة بالأخطاء المفاهيمية الرياضية كانوا من أمريكا، إنكلترا، وتركيا على الترتيب. وكانت الأوراق التابعة لباحثين من أمريكا هي الأوراق الأكثر استشهادًا. أما أكثر المواضيع الفرعية شيوعًا والمرتبطة بالأخطاء المفاهيمية الرياضية فقد كانت الأخطاء المفاهيمية الشائعة، إعادة بناء فهم المفاهيم الرياضية، والتعلم والتعليم المرتبط بالاقترانات والرسوم البيانية. في حين كانت أكثر الكلمات المفتاحية استخدامًا هي الطلبة، التربية، والتدريس. وقد أظهر التحليل النوعي لأهم عشرة أوراق بحثية تبعًا لعدد الاستشهادات وجود (23) خطأ مفاهيمي رياضي شائع، مصنفة إلى أربعة مجالات هي أخطاء مفاهيمية رياضية عامة، أخطاء مفاهيمية رياضيه جبرية، أخطاء مفاهيمية رياضية مثلثية، وأخطاء مفاهيمية رياضية مرتبطة بالتفاضل والتكامل. توصي هذه الدراسة بأن يقوم الباحثون بتوسيع نطاق أبحاثهم في التحليل الببليومتري في مجالات أخرى، نظرًا إلى النتائج القيّمة التي حصلنا عليها بعد استخدامه في مجال الرياضيات.

الكلمات المفتاحية: تحليل ببليومتري، الأخطاء المفاهيمية الرياضية، الأبحاث العلمية.

CHAPTER ONE

Background and Significance of the Study

1.1. Introduction

The concepts serve as building blocks of knowledge, allowing students to relate information to one another or to distinguish it from other information. Consequently, they play an integral role in learning and thinking. Meaningful learning occurs when you fully comprehend these concepts and establish connections between them (Julius et al., 2021).

As students experience mathematic concepts, they process information in ways that may be incorrect; some may memorize directly, while others may interpret incorrectly, all of which can cause misconceptions (Kadarisma et al., 2020).

A misconception is a mistake in understanding the concept or in interpreting its meaning (Ay, 2017). A misconception encountered by a student during mathematics classes can have a lasting effect on their understanding of mathematics in the future. According to Kadarisma (2016), Mathematical concepts are not isolated but are interconnected, so one mistake in a basic concept can lead to another mistake.

Mathematical education, as well as other disciplines, is the subject of many national and international scientific research, reviews, and studies. In response to scientific developments, the content of mathematics courses has varied, exposing various perspectives on how mathematics education can meet the needs of a developing society. A trend analysis of recent studies conducted in mathematics education will shed light on short-, middle-, and long-term studies, as well as guide researchers and educators in scientific debates. (Ozkaya, 2018)

Bibliometric analysis is an approach based on analysis that is often used in extensive scientific research to examine and analyse data. The bibliometric analysis allows the categorization of research development by articles, authors, and journals. Researchers use bibliometric analysis for a variety of research purposes, including describing the field of research, journal performance, research object collaboration, and exploring other developments in research. (Sreylak et al., 2022)

The development of mathematics education from a bibliometric perspective has been the main focus of discussion within the educational field. There is therefore a need for updated data derived from bibliometric analysis in mathematics education. In contrast to systematic review papers, bibliometric research involves analyzing published articles to identify global patterns within a specific academic field (Phan et al. 2022).

Bibliometric citation analysis facilitates quantitative evaluation of prominent journal titles, keywords, and the flow of publications in the academic context. Moreover, it provides valuable insights into the academic community by visualizing interactions between authors from diverse universities, institutions, and countries. (Julius et al., 2021)

1.2. Problem Statement

Many students pursue higher education after high school, but more than 40% discover that they lack the skills to succeed in college courses, particularly mathematics. Even though many math concepts are covered in middle and high school, some students still face gaps in their preparation (Lee & Boyadzhiev, 2020). Often, misconceptions about math can negatively affect students' performance, cause them to lose motivation for math, and affect their willingness to dedicate more time to the subject.

There are many findings on mathematics education, especially on mathematics misconceptions. Much research undertaken in bibliometric analysis in the field of sciences. For instance, only a limited number of studies carried out bibliometric analysis to map the global research landscape in the field of mathematics education. For example, Julius et al. (2021) reported a bibliometrics analysis of research in Mathematics education using the Scopus database. The researcher realized that there is a need for a bibliometric analysis to analyse all the articles on mathematics misconceptions from various studies.

The scope and complexity of the research issues are broad since mathematics education articles examine all concepts and domains related to mathematics. The bibliometric analysis concerning mathematics education suggested carrying out additional studies in this field while there are too many papers on this topic to be examined by a limited number of studies (Julius et al. 2021).

1.3. Study Purpose

The purpose of this study is to analyse scientific articles on mathematics misconceptions using bibliometric analysis from 1947-2023.

1.4. Study Questions

To achieve this goal, the study aims to answer questions related to two main sub-purposes:

A) Performance Analysis and benchmarking:

1) What is the journal publications' trend in mathematical misconception over its lifetime?
2) What is the journal citations' trend in mathematical misconceptions and what are the most cited papers?
3) Who are the most productive and influential institutions, and countries in mathematical misconceptions publication?

B) Content analysis and road map for future research

1) What are the main thematic patterns in mathematical misconceptions?
2) What are the most frequent misconceptions used in the studies published in the mathematics misconceptions research area?

1.5. Significance of the Study

As an actual effort to help in mathematics education, this research seeks to dig deeper into the worldwide research trend in mathematics misconceptions, therefore, the results of this study are expected to provide recommendations for further research and become an evaluation in learning mathematics. Mathematics teachers can use this study to avoid making these mistakes when teaching, training, and tutoring students. Curriculum developers can also create mathematics content in a way that reduces the likelihood of students making these mistakes.

Bibliometric analysis improves our understanding of science, which helps create knowledge in a variety of fields, including education. This study has significant importance for academics who are interested in bibliometric analysis. It is also beneficial for mathematics teachers to examine research regarding misconceptions in mathematics.

1.6. Definitions of Key Terms

The study includes some concepts and terms that need to be clarified as follows:

Mathematic Misconception: Kshetree et al. (2021) defined mathematics misconceptions as an individual's understanding of a concept that deviates from the intended meaning and comprehension in mathematics. The development of misconceptions in students can be attributed to their previous inappropriate learning.

Procedurally, the main theme in mathematical misconceptions is analysed across different fields including numbers, geometry, algebra, statistics, and measurements.

Bibliometric analysis: According to Norton (2000) bibliometric is the measurement of texts and information. Furthermore, (Daim et al., 2006) state that examining, organizing, and analyzing a large amount of historical data can be beneficial in finding hidden patterns that could aid researchers in making decisions.

Procedurally, a bibliometric analysis is used in this research to analyse bibliometric data from 1947 to 2023 on scientific articles on mathematics misconceptions.

1.7. Study Limitations and Delimitations

Study Limitations

Bibliometric analysis will analyse scientific publications indexed in the Scopus database between 1947 and 2023 on mathematics misconceptions. The sources of the articles are limited to those published in journals or conferences excluding books and book chapters.

Study Delimitations

The techniques chosen and the choices made at each stage of the bibliometric analysis are crucial because they affect the findings and conclusions made from the investigation. Moreover, the bibliometric data from scientific databases like Scopus are not created specifically for bibliometric analysis, therefore, they may contain minor errors.

CHAPTER TWO

Theoretical Framework and Review of the Literature

This chapter discusses the theoretical framework, as well as a review of relevant theoretical literature and prior research.

2.1. Theoretical Framework

Mathematics education is a fundamental and universal subject in education that encompasses various aspects, such as curriculum development, pedagogical approaches, assessment, and the overall improvement of mathematical knowledge and skills in students.

The field of mathematics emphasizes developing problem-solving skills, a crucial skill throughout students' lives; techniques and approaches gained at school have an important impact. Additionally, mathematics allows for the creation of meaningful problems and the development of new dialogs. It is through abstractions that children learn to understand relationships, recognize structure, reason about concepts, and debate a statement's truth/falsity (Mishara, 2020).

Considering the growing focus on STEM (Science, technology, engineering, and mathematics) education and careers, it’s critical to evaluate students’ understanding of the fundamentals of science and mathematics throughout their education and to determine any misconceptions, errors, or misunderstandings that may still exist. A lack of foundational understanding at earlier ages is linked to misconceptions, errors, and misunderstandings in higher grade levels. This is a crucial understanding for many stakeholders in science and mathematics education, including classroom teachers, teacher educators, policymakers, and researchers. (Neidorf et al., 2020)

Ersoy (2006) believes concept learning is important in mathematics, leading to numerous misconceptions. The interconnected nature of mathematical concepts and the spiral structure of mathematics curricula make defining any concept independently difficult. Therefore, students with misconceptions from previous topics may incorporate new misconceptions into the previous ones.

NCTM (2000) has confirmed that conceptual knowledge plays an essential role in the acquisition of knowledge by learners. A procedure, that assists the learner in implementing algorithms and mathematical skills, as well as determining the extent to which the procedures followed by the learner are correct.

A misconception is part of a larger system of concepts used by individuals to interpret daily experiences and understand them. Students establish connections between new knowledge and their previous knowledge when they encounter new knowledge. In case the new input conflicts with the existing knowledge, the existing knowledge needs to be restructured. Unintentionally, students may develop misconceptions through explanations, problem-solving, or presenting evidence based on incorrect reasoning. Frequently, these misconceptions blend with others and errors, leading to a cycle of misperceptions (Kurtulus & Tatar 2021).

Misconceptions are just as common as any other phenomenon. Almost any concept has the potential to be misunderstood. However, it’s not just an error; it may occur due to any mistake, but misconceptions may arise due to an inaccurate and incorrect understanding of prior knowledge. Misconceptions occur in computations, algebra, geometry, trigonometry, linear equations, quadratics equations, similar triangle relations, functions, and almost any other area of mathematics. (McDonald, 2010)

Mathematics concepts must be taught by teachers with accurate knowledge. Students should be protected from being misled by using appropriate methods and techniques in learning environments. Misconceptions can be prevented by educators proactively if they are familiar with them. Through scientific approaches and models, students can be assisted in reorganizing and incorporating information by recognizing their misconceptions, creating a discussion that encourages them to address these misconceptions, and addressing misconceptions via discussions and discussions. (Unver & Elci 2022).

The concept of bibliometrics was introduced by Pritchard (1969) as a new way to conduct reviews. It has been used in a variety of research topics, including mathematics (Phan et al. 2022). The bibliometric analysis is a convenient way to examine the development of a research domain, including the topics and authors, based on social, intellectual, and conceptual factors. This method has been applied to a variety of fields, including strategic management, corporate social responsibility, medicine, and corporate universities (Singh et al., 2020).

Many details about a source document are included in a bibliographic data collection, such as the author, the nation, the keywords, the institution, the language, the publication source, the year of publication, the references mentioned, the reference sources, and the subject categories. After these data are gathered, a bibliometric analysis is carried out to look at the connections between citations in scholarly journals. Publications are usually examined in terms of the number of times they are cited elsewhere as part of the citation analysis process. Comparing papers can also be done using co-citation ratios. (Drijvers et al., 2020).

Science mapping, which looks at the connections between researchers, and performance analysis, which assesses the contributions of researchers, are the two primary subcategories of bibliometric analysis. The next subsections provide explanations of science mapping approach and performance analysis (Donthu et al., 2021).

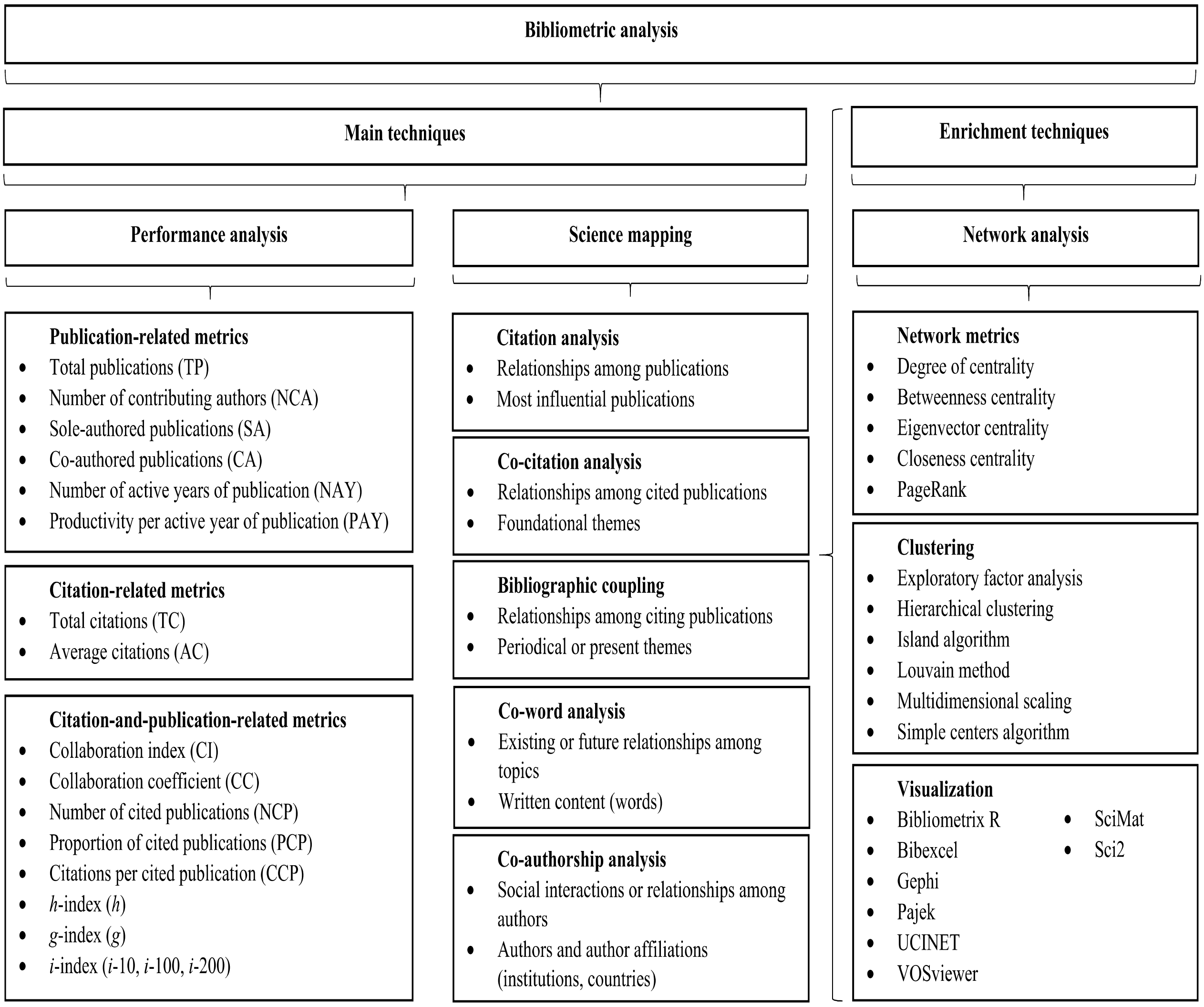


Figure (1): Toolbox for the bibliometric analysis (Donthu et al., 2021)

Performance analysis

Performance analysis assess the contributions that different research entities have made to a certain topic. This type of analysis, which is characterized by its descriptive nature, is fundamental to bibliometric studies. Science mapping is commonly featured in reviews, even when there is no science mapping involved, as it is common for reviews to present in-depth analyses of different research constituents (such as authors, institutions, countries, and journals). In a sense, this presentation is analogous to the background or profile of participants in empirical research (Ramos-Rodriguez & Ruiz-Navarro, 2004), but with a more analytical approach.

Science mapping

The science mapping method examines the relationships between research elements, focusing on their intellectual interactions and structural relationships. A science map is created by analyzing citations, co-citations, bibliographic couplings, co-word analysis, and coauthorships. In combination with network analysis, these techniques illustrate both the bibliometric structure and the intellectual framework of the research field (Cobo et al. 2011).

Enrichment techniques

To enhance the results of applied analysis techniques in bibliometric studies, this section introduces additional components that can be incorporated into fundamental bibliometric analysis methods. There are three enhancement avenues based on network analysis: network metrics, clustering, and visualization.

2.2. Literature Review

The researcher reviewed many earlier studies on bibliometric analysis and mathematics education; the oldest to newest are listed below.

Behrens & Luksch (2011) conducted a bibliometric analysis of the mathematics literature published between 1868 and 2008, using data from the Zentralblatt MATH database. The results showed that there was an increase in the rate of publications per year that reflects the growth of the mathematics community, author productivity has only changed slightly since the year 1870, and the percentage of single-authored papers has declined from over 95% before 1930 to about 30% today.

Jimenez-Fanjul et al. (2013) analysed the production of mathematics education in the Social Sciences Citation Index (SSCI) of the Web of Science. The study examined four SSCI-indexed journals on Mathematics Education, comprising 1356 articles. A study was conducted to identify the most important institutional collaboration networks. According to the results, Research in Latin America ranks slightly higher than research in Europe.

According to Ersozlu & Karakus (2018), the study examined publications related to math anxiety and contributed to its development. Bibliometrics was used to analyse 537 publications that were published in the Web of Science between 2000 and 2018. The findings of this study suggest that math anxiety has often been investigated in connection with motivation, self-efficacy, self-concept, and anxiety. In this discipline, journals such as Frontiers in Psychology and Learning and Individual Differences are crucial, and the University of Chicago is a major player.

Ozkaya (2018) analysed a bibliometric analysis of the scientific research published in mathematics education between 1980 and 2018 from the source scanned in the Web of Science, applying an objective approach derived from statistical analysis to examine the structure of scientific knowledge and communication in the field, a total of 9841 scientific research were found and analysed using Citespace software. The results showed that the publications involving mathematics education increased from 1980 to 2018, the most productive countries are the United States, England, and Turkey, the highest numbers of citation bursts come from the US, Turkey, and Malaysia, and the words that are commonly used in mathematics education research are mathematics, education, student, and achievement.

A study by Ramirez & Devesa (2019) examines 5633 publications that are linked to mathematics education research and are listed in Scopus. According to the findings, scientific productivity has increased exponentially, collaboration has increased over individual work, cited references have had a higher impact over the last 20 years, international collaboration has increased, publications in some internationally recognized journals tend to focus on a smaller number of researchers, and 17 invisible colleagues have emerged that represent scientific communities that share a common interest.

Domenech et al. (2019) presented a bibliometric overview of academic research in STEM education. Visualization of similarities (VOS) viewer software was used to visualize the bibliographic data. The findings suggested a rise in efforts dedicated to the topic in recent years; however, the rate of citation growth had not paralleled this increase. Furthermore, upon examining journals and research domains, STEM education research exhibits a broad range of diversity.

Ha et al. (2020) evaluated the scientific results of STEM education in the Association of Southeast Asian Nations (ASEAN) region, as indexed in the Scopus database from 2000-2019. A total of 175 publications were analysed from the Scopus database using Bibliometrix, VOSviewer and Microsoft Excel. The study highlights that published work represents 67.43% of all scientific output in this field over the past three years.

A bibliometric analysis of research on mathematics education from 1980 to 2020 aimed to present scientific information on the distribution pattern of journals in mathematics education, the most active authors, nations, organizations, current research topics, potential areas of international collaboration, and research directions in mathematics education. There were 12670 articles in the Scopus database, and based on the cumulative articles' performance indicator, there is likely to be an increase in publications. Problem-solving, professional development of teachers, and curriculum development were the primary areas of interest for the discipline. Mathematical topics such as algebra, proof, calculus, technology, geometry, and modeling received the most attention (Julius et al., 2021).

The study by Kurtulus, and Tatar (2021) aimed to conduct a bibliometric analysis of the published articles related to science misconceptions. To achieve this, a bibliometric analysis involving 859 articles published in the fields of biology, chemistry, and physics education between 1986 and 2019 was carried out, the scanned articles were analysed using R-Studio program. The results indicate that science misconceptions have increased since 2010. There have been the most publications on this topic in the Journal of Science Education, while David F. Treagust has published the most articles on this topic. The article published in 2012 by Furtak was the most cited. In addition, the findings show that Korea and China are the country's most open to collaboration, and that 'science' and 'students' are frequently used keywords.

Based on bibliometric analysis of 12,272 publications from WOS database, Djeki et al. (2022) examined collaborations between authors, universities, countries, and reference papers, and the African contribution to the field of e-learning using VOSviewer program, The results of the study, authors, universities, and countries involved in e-learning rarely collaborate, and the COVID-19 pandemic has had a substantial impact on e-learning. According to the results, the United States, Spain, England, and China are the most productive countries in e-learning.

A literature review was conducted using bibliometric analysis by Sreylak et al. (2022) to describe the trend and development of research about mathematics concepts in elementary schools in Indonesia. The bibliometric analysis outcomes from the relevant articles covered areas such as the understanding of mathematical concepts among elementary school students, knowledge of preservice students and teachers regarding these concepts, and possible ways of improving students' grasp of mathematical concepts at the elementary level.

The bibliometric study conducted by Muhammad et al. (2023) authors revealed the research focus linked to the innovation of Discovery Learning in mathematics education. Based on data gathered from Scopus, there were greater publications in mathematics learning between 2017 and 2023 that included Discovery Learning. The 2017 publication, which received 41 citations, showed a strong trend of citations. Indonesia has made a significant contribution to the mathematical study conducted by Discovery Learning.

2.3. Comments on Previous Studies

Numerous bibliometric studies have been carried out in the field of education, such as the study by Djeki et al. (2022) that discussed e-learning collaborations, Bibliometric analysis has rarely been used in mathematics education to map research trends worldwide. As an example, Muhammad et al. (2023) investigated Discovery Learning in mathematics while Behrens & Luksch (2011), Ramirez & Devesa (2019), Ozkaya (2018), Jimenez-Fanjul et al. (2013) and Julius et al. (2021) reviewed publications in mathematics education. Meanwhile, Sreylak et al. (2022) conducted literature review research on mathematics concepts. Ersozlu & Karakus (2018) examined publications on mathematics anxiety.

Ha et al. (2020) and Domenech et al. (2019) conducted bibliometric analyses of STEM education, whereas the purpose of this study is to examine mathematical misconceptions. Furthermore, this study focuses on misconceptions about math education. In a different field, Kurtulus & Tatar (2021) conducted a bibliometric analysis of articles concerning misconceptions.

CHAPTER THREE
Methodology and Procedures

This chapter includes a description of the study methodology, the study population, and its sample, as well as a description of data collection, and the processing data used to draw conclusions.

3.1. Study Methodology

This study follows the general bibliometric analysis workflow, which consists of five stages: Study Design, Data Collection, Data Analysis, Data Visualization, and Interpretation (Börner et al., 2003; Zupic & Cater, 2014). Bibliometric methods were used to establish global research patterns in mathematics misconceptions. Systematic analysis (Qualitative Approach) of published articles was used to examine the common misconceptions identified by bibliometric analysis in the top ten articles. An article's impact is assessed by its citations, along with other related articles. The distribution of publications over time, journals, countries, institutions, author performances, and the main subjects receiving the most attention and their shifts in focus over time were quantified in order to chart the trends on the current subject.

3.2. Sampling Method

According to the Scopus database, the data for the study is based on English-language published articles in the period of 1947-2023 that contain the keyword "mathematics misconception, math concepts, errors in math". The articles were published in various journals between 1947 and 2023. In the analysis of the database, books, book chapters, review articles, editorial materials, and letters are excluded. Based on the scanning, it was determined that the first article on the subject matter was published in 1947; therefore, 1947 is considered to be the start date. As 2024 has not yet ended, it was thought that including articles published in 2024 so far might affect the study's results; therefore, these articles were excluded from the study's sample.

3.3. Data Collection

Using the keyword "mathematics misconception, math concepts, math errors ", 750 publications were retrieved from the database. Several search limitations were imposed in keeping with the study's purpose, such as a journal article, a subject area, a language, and a time period, and 525 articles will be used in the analysis. An analysis of this sample of articles will be conducted to determine the distribution of articles; the average citation scores; the list of the journals that published the highest number of related articles; the list of the authors that had published the highest number of related articles; the citation burst scores of the authors; the scientific productivity of the countries of the authors; the articles that were cited at the highest rates; collaboration networks; and their patterns that were obtained through text mining methods of word cloud and word tree. The flowchart in Figure 1 illustrates the strategy for collecting data and searching.

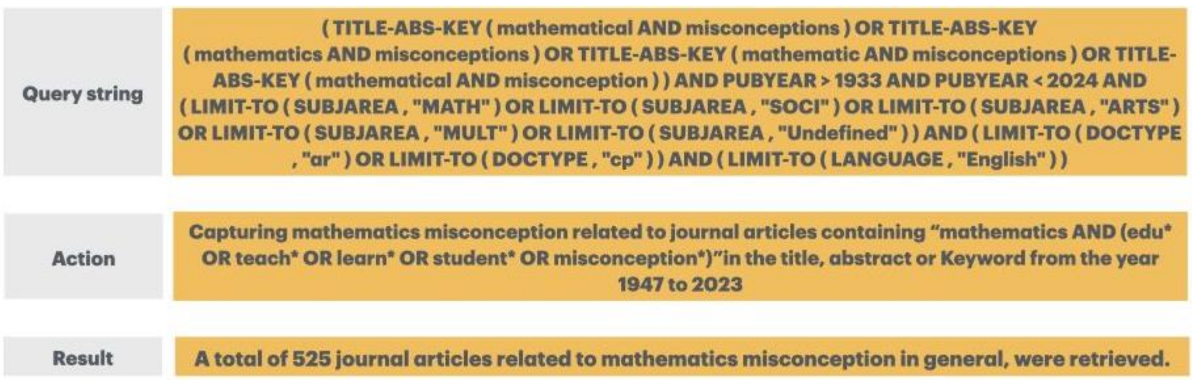


Figure (2): Flowchart of data collection and search strategy

3.4. Data Analysis

The scanned articles were analysed using R-Studio and VOSviewer programs within the scope of the research. R program was accessed at https://cran.r-project.org/, which is the official storage website of many bibliometric analysis packages. These package programs for bibliometric analyses are quite beneficial in quantitative research (Aria & Cuccurullo, 2017). Since R program provides more variety of results with enriched details, R program will be chosen for the bibliometric analyses conducted in the present research.

The data file used for the study was formed through Scopus based on the criteria identified for the research and following specific steps for the selection of the articles. The first is to select “export”, next “other file formats”, then “records from (1- 500)”, and finally “record content (Full Record and Cited References)”. The data file for the study after completing the selection steps consisted of 525 articles in total. To analyse the articles, first, the “bibliometrix” package in the R program will be downloaded and activated for the analyses. Then, R- Studio program was directed through a web address to the bibliometric analysis page. Here, the “Plain text” file is saved into a data segment where the analyses of the study will be conducted. Finally, the most common mathematics misconceptions between 1947 and 2023 were extracted by conducting a qualitative content analysis on the top ten papers, which were determined by the bibliometric analysis.

3.5. Study Procedures

The study followed the following procedures.

- Review of educational literature and previous studies.
- Define the aims and scope of the bibliometric study.
    - What are the aims and scope of the study?
    - Is the scope of the study large enough to warrant the use of bibliometric analysis?
- Choose the data for bibliometric analysis.

- What bibliometric analysis techniques should be chosen to meet the aims and the scope of the study?

- Collect the data for bibliometric analysis.
  - Do the search terms exemplify the scope of the study?
  - Is the coverage of the database adequate for the study?
  - Is the data free of errors such as duplicates and erroneous entries?
  - Does the final dataset fulfil the requirements of bibliometric analysis techniques chosen for the study?
- Run the bibliometric analysis and report the findings.
  - Can the bibliometric summary be easily understood by readers?
  - Does the writing align with the bibliometric summary presented?
  - Does the writing explain the peculiarities and implications of the bibliometric summary?
  - Does the writing align with the target outlet for publication?
- Discussing the results and writing recommendations.

CHAPTER FOUR

Findings and Results

This chapter summarised the study's findings by responding to the study's specified study questions, and the following is a summary of the study's findings.

4.1. The findings related to Performance Analysis and benchmarking of mathematics misconception.

4.1.1. Overview of Mathematical Misconceptions

Table 1 provides the main information regarding the collection in the field of mathematical misconceptions encompassing various aspects such as document count, author appearances, and document types. It begins by stating that there are 525 documents in total, sourced from 252 different sources including journals and books. Additionally, the dataset contains 823 identified keywords Plus (ID) and 1303 author-specified keywords, reflecting the breadth of topics covered.

The timeframe of the data spans from 1947 to 2023, indicating a long period of study or analysis. Notably, each document has received an average of 18.23 citations, suggesting a significant impact within the scholarly community.

Regarding authorship, the dataset comprises contributions from 1148 authors. Among these, 129 authors have solely authored documents, accounting for 133 single-authored documents. On average, each author has contributed 0.46 documents, while each document has received contributions from 2.19 authors. Furthermore, each document, on average, has 2.45 co-authors, indicating a collaborative research environment.

In terms of document types, the majority consists of articles (447), followed by conference papers (78). This breakdown provides insights into the distribution of scholarly output within the dataset, with articles being the predominant form of publication in the field of mathematical misconceptions.

Table (1): Overview of Mathematical Misconceptions

Description	Results
Documents	525
Sources (Journals)	252
Keywords Plus (ID)	823
Author's Keywords (DE) 5	1303
Period	1947 - 2023
Average citations per document	18.23
Authors	1148
Author Appearances	
Authors of single-authored documents	129
Single-authored documents	133
Documents per Author	0.46
Authors per Document	2.19
Co-Authors per Document	2.45
Document types.	
Article	447
Conference Paper	78

4.1.2. Publication Output and Growth Trend

Table 2 and Figure 3 describe the publication output and growth trend, with a detailed breakdown of publications and citation statistics across distinct periods, offering insights into the evolution of research impact over time. The analysis spans from 1947 to 2023, delineated into 5 intervals.

In the earliest period, from 1947 to 1962, there was a solitary publication, garnering 78 cumulative citations, with an average of 7 citations per paper. This suggests a modest but notable impact on that era.

Moving forward to the subsequent period, spanning 1963 to 1978, the number of publications increased to three, and a total citation of 102, accompanied by a rise in cumulative citations to 180, with an average of 46 citations per paper. As a result, the output and influence of research are gradually increasing.

From 1979 till1994, both publications and citations increased substantially, as shown in the table. 30 publications and 402 citations accumulated a total of 582 citations during this period, with an average of 542.1 citations per paper. The considerable increase in research activity reflects a growing interest and impact in mathematical misconceptions among scholarly communities.

Between 1995 and 2010, 114 publications amassed 330 citations, 912 cumulatively, and an average of 736.2 citations per paper. Over this period, research output and citation rates have significantly increased, indicating an increasingly robust and influential body of work.

Furthermore, from 2011 to 2023, the table shows a remarkable expansion in research activity, which is illustrated by 377 publications with 104 citations and 1016 citations cumulatively. Citations per paper during this period dropped significantly in comparison with previous years, indicating a proliferation of publications of varying degrees of impact.

Table (2): The number of publications in the field of mathematical misconceptions and its growth trend

Year	TP*	CC**	NCP ***	TC ****	C/P *****
1947-1962	1	78	1	78	7
1963-1978	3	180	3	102	46
1979-1994	30	582	30	402	542.1
1995-2010	114	912	114	330	736.2
2011-2023	377	1016	377	104	130.71

*TP: Total Publications ** CC: Cumulative Citation *** NCP: Number of Cited Publications **** TC: Total Citation *****C/P: Average Citation Per

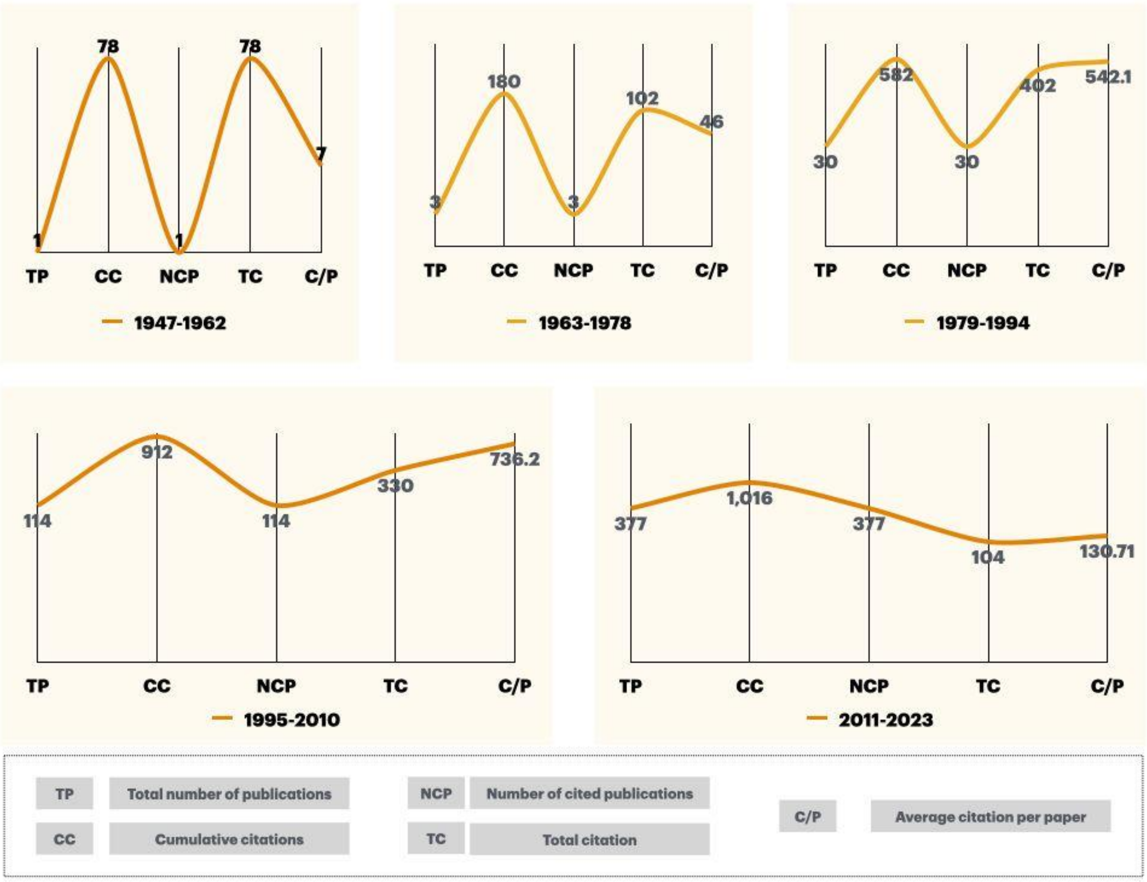


Figure (3): Publications on mathematical misconceptions and their growth trend

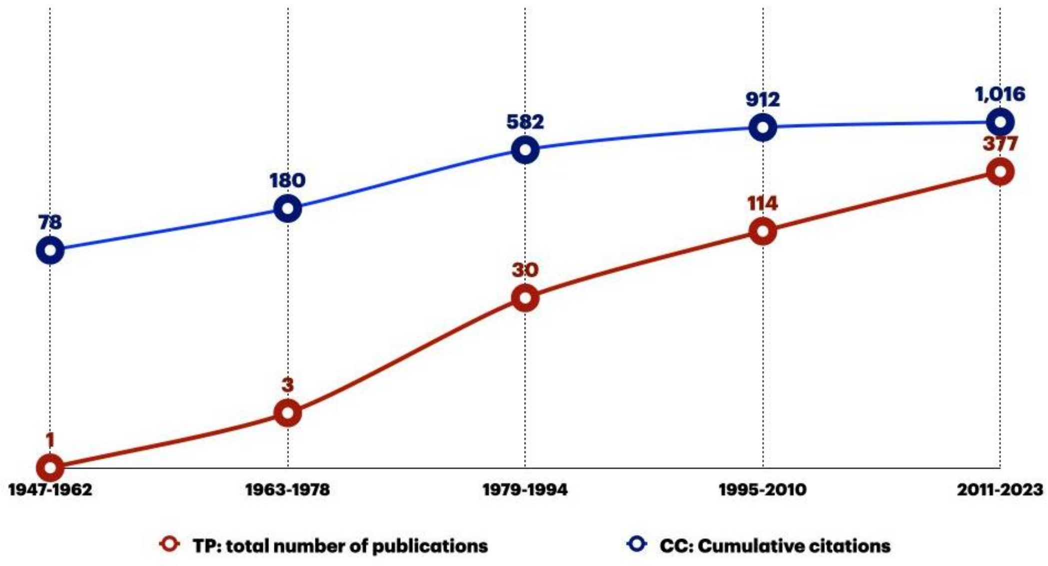


Figure (4): The number of publications and their cumulative citation values in the field of mathematical misconceptions in terms of years

Over several periods spanning from 1947 to 2023, figure 4 shows the annual number of publications in the field of mathematical misconceptions and their cumulative citation values. Two-line graphs are included, one showing the number of publications per year and the other showing cumulative citations.

Scholarly attention to mathematical misconceptions has grown over the past decades, both in terms of publications and citation values.

There were relatively few publications and citations in the field in earlier periods, from 1947 to 1994. Between 1979 and 2010, there was a significant increase in both publication numbers and cumulative citations during the third and fourth intervals.

There's a notable increase in publication outputs and cumulative citations in the second interval, covering the years 2011 to 2023. A rise in interest and support in eliminating mathematical misconceptions is anticipated during this time due to developments in educational psychology, technology, and pedagogical approaches.

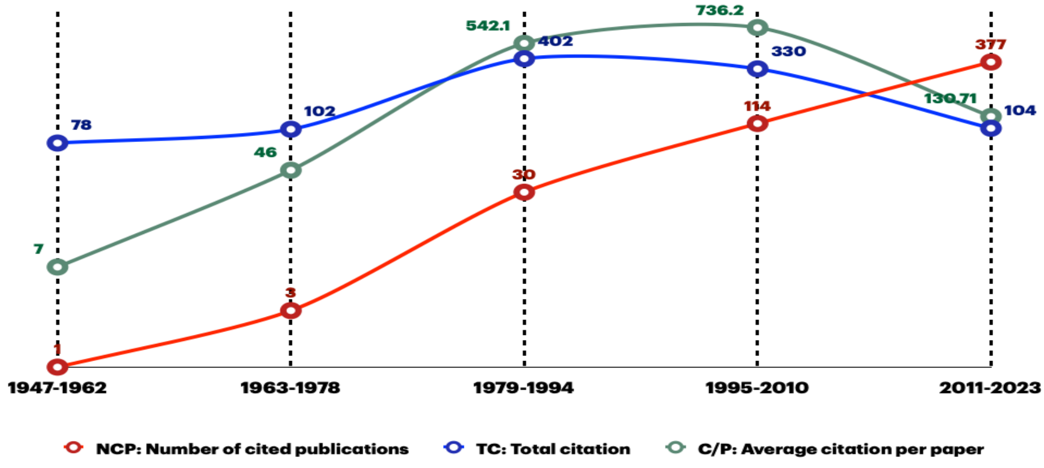


Figure (5): Number of cited publications, total citations, and average citation per paper in the field of mathematical misconceptions

Figure 5 displays three-line graphs showing various indicators from 1947 to 2023. The graph shows the changes over the designated intervals in citation patterns, average citation rates, and the total number of cited articles. Offering insightful information about academic research and citation patterns in the area of mathematical misconceptions.

The first line, represented in red, shows how many publications have been cited. There is only one referenced article in the first interval, and this number progressively rises to 377 in the last interval, indicating a notable increase in cited publications.

The second line, represented in blue, displays the total citations for each interval. It shows fluctuating values over time, with a peak of 402 citations in the interval from 1979 to 1994 and a notable decrease to 104 citations in the last interval from 2011 to 2023.

The third line, depicted in green, represents the average citation per paper. It demonstrates a dramatic increase from 7 in the first interval to 736.2 in the interval from 1995 to 2010, indicating a substantial rise in the average citation rate per paper over time. However, there's a noticeable decrease to 130.71 in the last interval.

4.1.3. Country Production over time

Table 3 provides the number of articles published within different regions across five distinct time intervals. From 1947 to 1962, only one article was recorded from USA, with no contributions from the UK, South Africa, Indonesia, or Turkey. In the subsequent period, spanning from 1963 to 1978, the number of articles increased, particularly in the USA, which contributed five articles, while the UK contributed one. However, South Africa, Indonesia, and Turkey had no recorded publications during this period.

The trend of growth becomes more pronounced in the following years, from 1979 to 1994, with a substantial increase in the number of articles published across all regions, particularly in the USA and the UK. South Africa, Indonesia, and Turkey still showed no publications during this time frame.

From 1995 to 2010, there was a significant increase in the volume of articles, especially from the USA, with contributions also emerging from the UK, South Africa, Indonesia, and Turkey. However, the number of articles from these countries remained relatively low compared to the USA.

Table (3): Country production over time in the field of mathematical misconceptions

Year	Number of Articles				
	USA	UK	South Africa	Indonesia	Turkey
1947-1962	1	0	0	0	0
1963-1978	5	1	0	0	0
1979-1994	84	30	0	0	0
1995-2010	989	276	37	5	26
2011-2023	34591	512	394	263	952

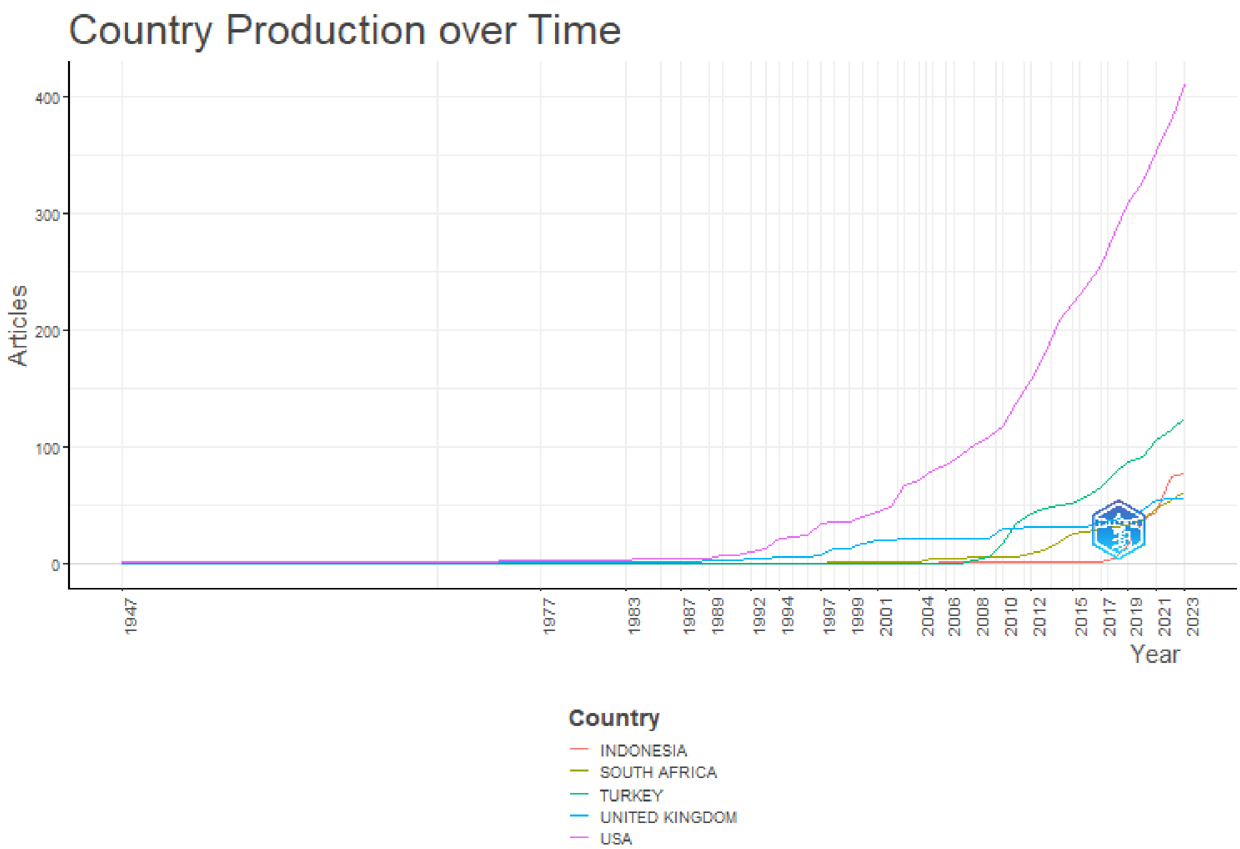


Figure (6): Number of articles (publications) by country and year in the field of mathematical misconceptions

As shown in Figure 6 the most recent period, spanning from 2011 to 2023, showcases a remarkable growth of publications of articles, particularly from the USA, with an exceptionally high number of contributions which reached 34591 articles. While the UK also maintained a notable presence with 512 articles. There was a notable increase in publications from South Africa, Indonesia, and Turkey, signifying a more diverse and globally distributed body of research output.

4.1.4. Contribution by Nations (impact of country)

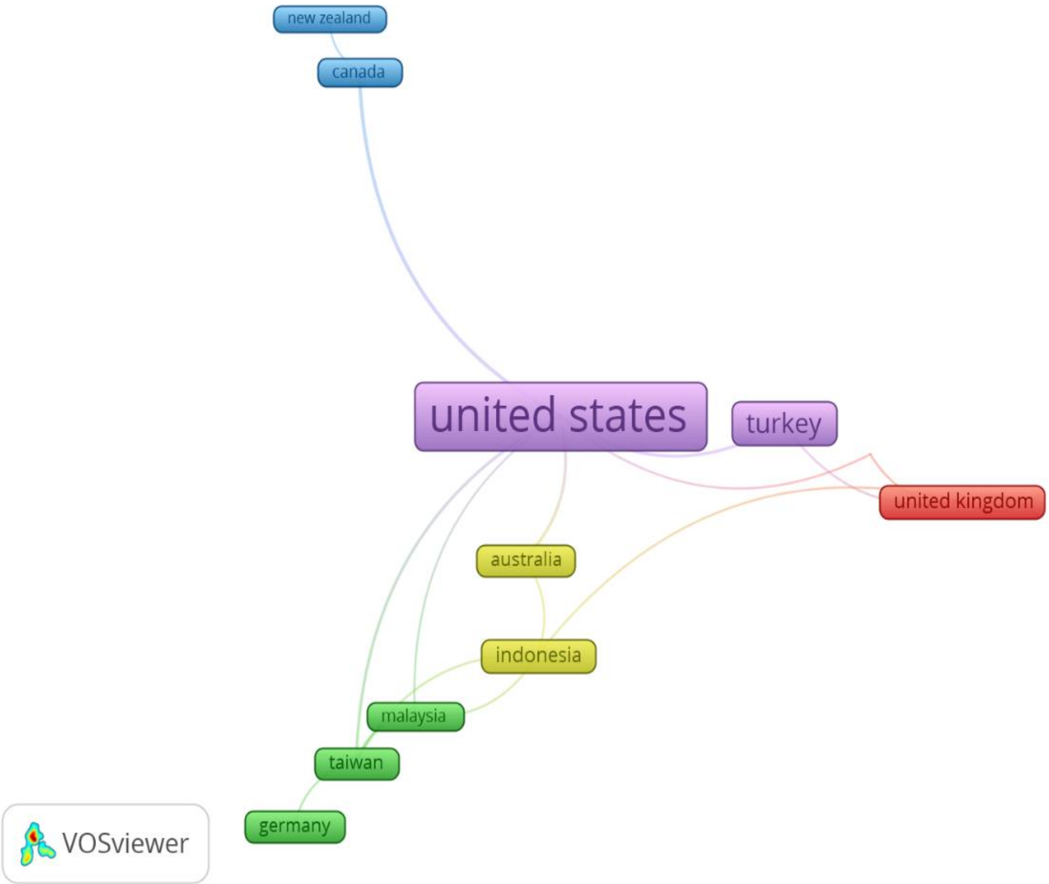


Figure (7): The academic cooperation on mathematical misconceptions research among countries in the world

The academic cooperation on mathematical misconceptions research among countries and other countries in the world is shown in Figure 7. The diagram was created by VOSviewer where the size of labels is proportional to the volume of documents for each country. The thickness of each line of connection indicates the corresponding degree of association. Colors symbolize 5 clusters with different degrees of relation.

As the diagram shows United States has the most publications and has research cooperation in the field of mathematical misconceptions with many other countries such as Australia, Turkey, Canada, Malaysia, South Africa, and Taiwan. The authors from the

United States cooperate mainly with authors from Canada in addition to a few other countries.

4.1.5. Contribution by Affiliations

Table 4 and Figure 8 present the 10 most productive affiliations publishing in the field of mathematical misconceptions, categorized by their affiliations. One notable aspect is the diversity of affiliations, including contributions from South Africa, the USA, the UK, Turkey, and Brueni indicating the global reach and collaboration within the field.

At the forefront, the University of the Witwatersrand in South Africa leads with nine articles, showcasing its significant contribution to academic discourse. Following closely behind is Arizona State University in the USA, boasting eight articles, indicative of its robust research endeavors. Meanwhile, the University of Manchester in the UK holds its ground with seven articles, highlighting its steady presence in the academic arena.

Similarly, Indiana University in the USA and Middle East Technical University in Turkey share the same number of articles, demonstrating their substantial research output. Notably, Boston University in the USA follows suit with six articles, underscoring its active engagement in scholarly pursuits. Further down the list, Comenius University in Slovakia and Karadeniz Technical University in Turkey each contribute five articles, emphasizing their commitment to advancing knowledge within their respective domains.

Towards the lower end of the spectrum, the University of Oklahoma in the USA and Universiti Brunei Darussalam in Brunei each present four articles, indicating their participation in academic research, albeit on a smaller scale compared to their counterparts.

Table (4): Top 10 most productive affiliations publishing in the field of mathematical misconceptions. (Contribution by Institutions)

Order	Affiliation	Country	Articles
1	University of the Witwatersrand	South Africa	9
2	Arizona State University	USA	8
3	University of Manchester	UK	7
4	Indiana University	USA	7
5	Middle East Technical University	Turkey	7
6	Boston University	USA	6
7	Comenius University	Slovakia	5
8	Karadeniz Technical University	Turkey	5
9	University of Okalahoma	USA	4
10	Universiti Brunei Darussalam	Brunei	4

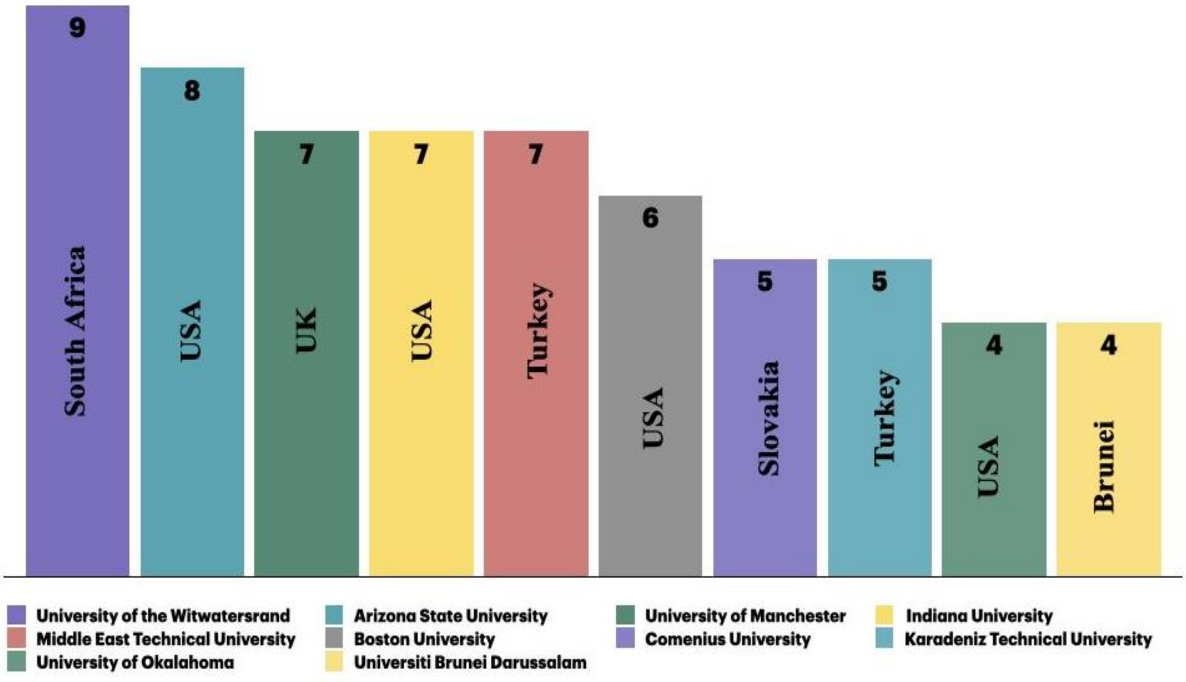


Figure (8): Top 10 most productive affiliations publishing in the field of mathematical misconceptions. (Contribution by Institutions)

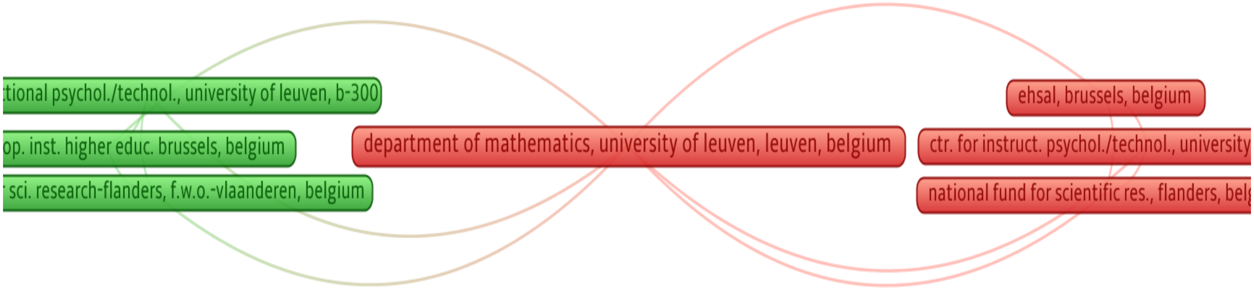


Figure (9): Cooperation between affiliations in the field of mathematical misconceptions

Figure 9 reveals the cooperation between affiliations in the field of mathematical misconceptions determined by VOSviewer where the size of labels is proportional to the volume of documents for each country. The thickness of each line of connection indicates the corresponding degree of association. Colors symbolize two clusters with different degrees of relation. The figure shows six groups of affiliations that have collaborations. However, collaborations in research on the topic are mainly from groups of authors working in 2 or 3 affiliations.

4.1.6. Contribution by document

Table 5 compiles a diverse array of academic papers, each offering unique insights and contributions to the mathematical misconceptions field. "Reliability in content analysis: some common misconceptions and recommendation" (2004), originating from the University of Pennsylvania in the USA, delves into content analysis and has garnered substantial attention with 1809 citations since its publication in Human Communication Research by Wiley-Blackwell. Similarly, "Misconceptions reconceived: a constructivist analysis of knowledge in transition" (1994), authored by Michigan State University in the USA and published in the Journal of Learning Sciences by Taylor & Francis, has made a notable impact with 1080 citations, reflecting its significance in educational discourse.

Another notable paper, "Functions, graphs, and graphing: tasks, learning, and teaching" (1990), stemming from the University of Pittsburgh in the USA and published in the Review of Education Research by SAGE, contributes to the understanding of educational processes and has garnered 589 citations. Meanwhile, "Enhancing prospective teachers' knowledge of children's conceptions: the case of division of fraction" (2000), originating from Tel-Aviv University in USA and published in the Journal of Research in Mathematics Education by the National Council of Teachers of Mathematics, has received 205 citations, underscoring its significance in the realm of mathematics education.

Further enriching the academic landscape, "There is more to discourse than meets the ears: looking at thinking as communicating to learn more about mathematical learning" (2001), authored by The University of Haifa in Palestine and published in Educational Studies in Mathematics by Springer Nature, has garnered 198 citations. These papers, along with others in the table, represent valuable contributions to their respective fields, shaping scholarly discourse and advancing knowledge worldwide.

Table (5): Mathematical misconceptions document contribution

Order	Document title	Year	citation	Journal	Publisher	affiliation	country
1	Reliability in content analysis: some common misconceptions and recommendation	2004	1809	Human communication research	Wiley-Blackwell	University of Pennsylvania	USA
2	Misconceptions reconceived: a constructivist analysis of knowledge in transition	1994	1080	Journal of Learning Sciences	Taylor & Francis	Michigan State University	USA
3	Functions, graphs, and graphing: tasks, learning, and teaching.	1990	589	Review of education research	SAGE	University of Pittsburgh	USA

Order	Document title	Year	citation	Journal	Publisher	affiliation	country
4	Enhancing prospective teachers' knowledge of children's conceptions: the case of division of fraction.	2000	205	Journal of research in mathematics education	National Council of Teachers of Mathematics	Tel-Aviv University	USA
5	There is more to discourse than meets the ears: looking at thinking as communicating to learn more about mathematical learning	2001	198	Educational studies in mathematics	Springer Nature	The University of Haifa	Palestine
6	The effectiveness of using incorrect examples to support learning about decimal magnitude	2012	187	Learning and instructions	Vanderbilt University	Elsevier	USA
7	Developing an assessment-centered e-learning system to improving student learning effectiveness	2014	120	Computers and education	National Tsing Hua University	Elsevier	Taiwan
8	An extensive analysis of preservice elementary teachers' knowledge of fractions	2008	112	American Educational Research Journal	Temple University	SAGE	USA
9	Seeing the complexity of standing to the side: instructional dialogues	2005	110	Cognition and Instruction	University of Pittsburgh	Taylor & Francis	USA
10	The irregular cutting-stock problem- a new procedure for deriving the no-fit polygon	2001	109	Computers and operations research	University of Southampton	Elsevier	UK

4.1.7. Impact of Journals according to documents

As shown in Table 6 and Figure 10, the top 10 most active journals in the field of mathematics misconceptions are listed with key metric information, including article counts, citation scores, H-index, Scopus quartiles, and publisher information.

There is an interesting diversity in the national origin of these publications, from the Netherlands and the UK to the USA and Germany, illustrating the international scope of mathematical education research. As indicated by their citation scores, H-index, and Scopus quartile ranking, the table also sheds light on the varying levels of impact and visibility of these journals.

Journals such as "Educational Studies in Mathematics" and "Journal of Mathematical Behavior" have higher citation scores and H-indices, indicating their influence in the field. Furthermore, Scopus' quartile rankings reveal these journals' relative standing within the academic community, with several among the prestigious first quartile.

Additionally, the information about publishers provides valuable context, as these publications are associated with well-known entities such as Springer Nature, Taylor & Francis, Elsevier, and Wiley-Blackwell.

Table (6): The 10 most active journals in mathematical misconceptions based on number of publications

Order	Journals	Nation of journals	No. of articles	Cite Score 2022*	H-Index*	Scopus quartile *	Publisher
1	Educational Studies in Mathematics	Netherland	444	4.7	13	First quartile	Springer Nature
2	International Journal of	UK	412	2.6	9	First quartile	Taylor & Francis

Order	Journals	Nation of journals	No. of articles	Cite Score 2022*	H-Index*	Scopus quartile *	Publisher
	Mathematical Education in Science and Technology						
3	Journal Of Mathematical Behavior	USA	180	2.7	8	First quartile	Elsevier
4	Primus	UK	153	1.2	4	Third quartile	Taylor & Francis
5	Lecture Notes in Computer Science	Germany	148	2.2	6	Third quartile	Springer Nature
6	Mathematics Education Research Journal	Netherland	128	3.7	3	First quartile	Springer Nature
7	School Science and Mathematics	USA	124	2.1	5	First quartile	Wiley-Blackwell
8	Proceeding - Frontiers in Education Conference, FIE	USA	118	1.1	3	Third quartile	Institute of Electrical and Electronics Engineers Inc.
9	International Journal of Science and Mathematics Education	Netherland	77	4.8	7	First quartile	Springer Nature
10	Mathematics Teaching-Research Journal	USA	68	2.7	3	Fourth quartile	Elsevier

* *According to Scopus (http://www.scopus.com) dated Month xx, year*

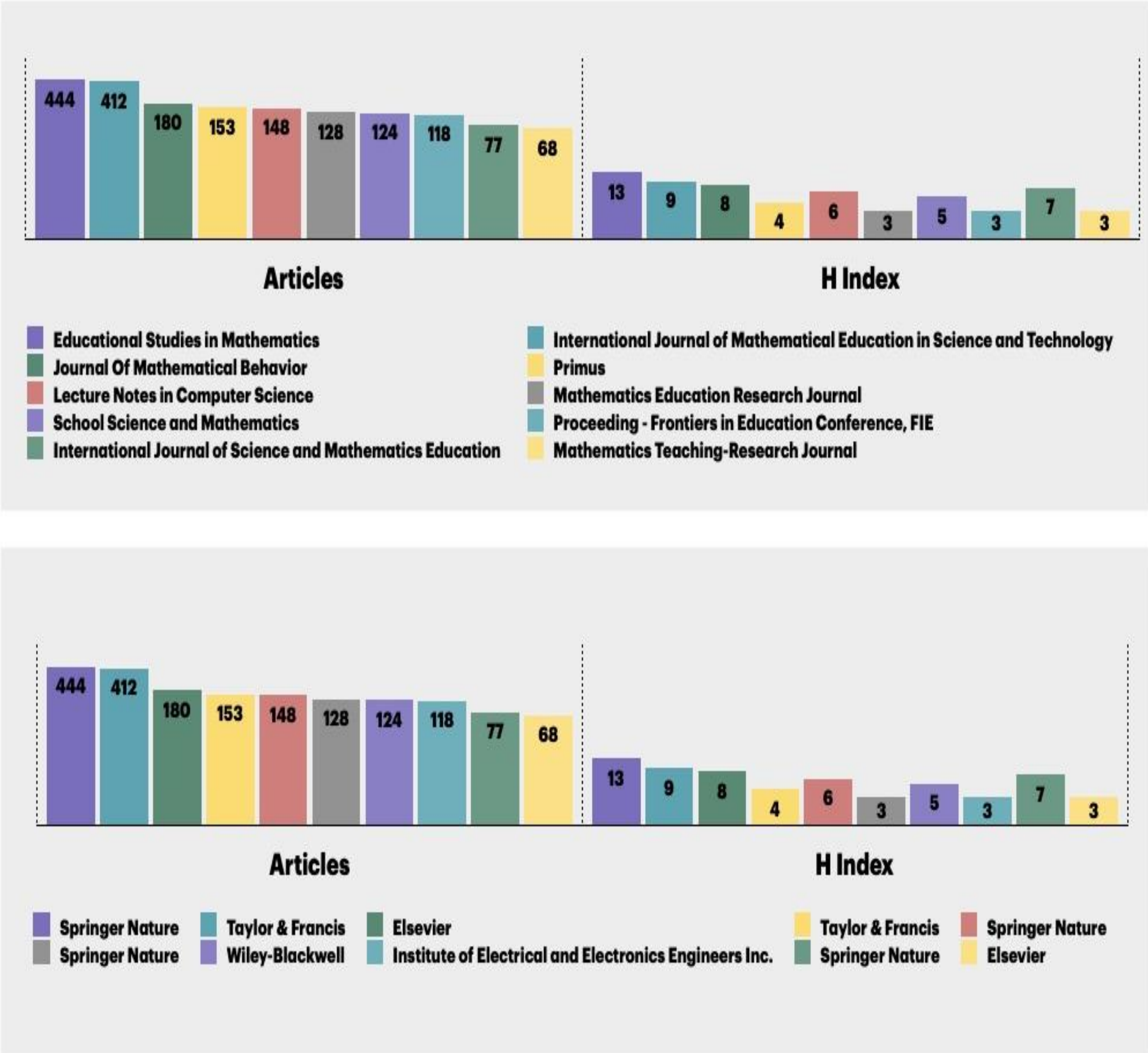


Figure (10): The 10 most active journals in mathematical misconceptions based on number of publications

4.1.8. Impact of Journals according to citation

An overview of ten influential journals in the field of education is presented in Table 7 and Figure 11, with a particular focus on mathematics education and learning sciences. Several of the journals are published by reputable publishers, such as Wiley-Blackwell, Taylor & Francis, Springer Nature, SAGE, and Elsevier, which emphasizes the quality and thoroughness of their editorial processes.

The table offers useful citation metrics for each journal, including citations, Cite Score, and H-Index. These indicators assess a journal's position and impact on the academic community. The journals "Review of Educational Research" and "Computers and Education" stand out for having a high number of citations and Cite Scores.

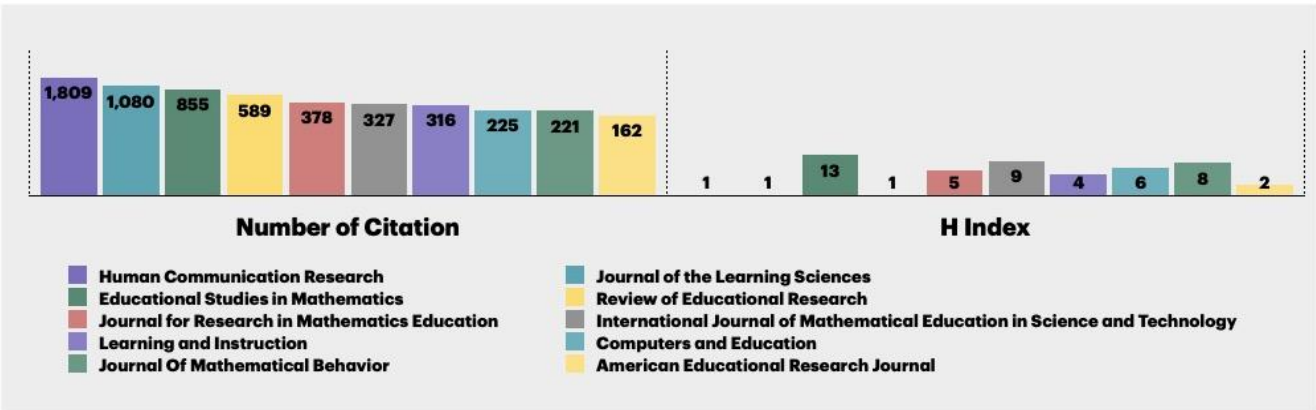
Furthermore, the Scopus quartile classification offers an idea of each journal's ranking in relation to its own subject. A number of the journals in the table are in the first quartile of publications for mathematics and education research, implying that they are widely respected.

The table additionally includes details on the nation of the journals, including a variety of publications from the US, the UK, and the Netherlands. The diversity within the field of mathematics emphasizes how important it is to collaborate and share research on mathematical misconceptions from a global viewpoint.

Table (7): The Top 10 Cited Journals for Articles on Misconceptions in Mathematics.

Order	Journals	Nation of journals	No. of Citations	Cite Score 2022*	H-Index*	Scopus quartile *	Publisher
1	Human Communication Research	USA	1809	6.9	1	First quartile	Wiley-Blackwell
2	Journal of the Learning Sciences	USA	1080	12	1	First quartile	Taylor & Francis
3	Educational Studies in Mathematics	Netherland	855	4.7	13	First quartile	Springer Nature
4	Review of Educational Research	USA	589	21.1	1	First quartile	SAGE
5	Journal for Research in Mathematics Education	USA	378	4.5	5	First quartile	National Council of Teachers of Mathematics

Order	Journals	Nation of journals	No. of Citations	Cite Score 2022*	H-Index*	Scopus quartile *	Publisher
6	International Journal of Mathematical Education in Science and Technology	UK	327	2.6	9	First quartile	Taylor & Francis
7	Learning and Instruction	UK	316	11.2	4	First quartile	Elsevier
8	Computers and Education	UK	225	23.8	6	First quartile	Elsevier
9	Journal Of Mathematical Behavior	USA	221	2.7	8	First quartile	Elsevier
10	American Educational Research Journal	USA	162	9.0	2	First quartile	SAGE



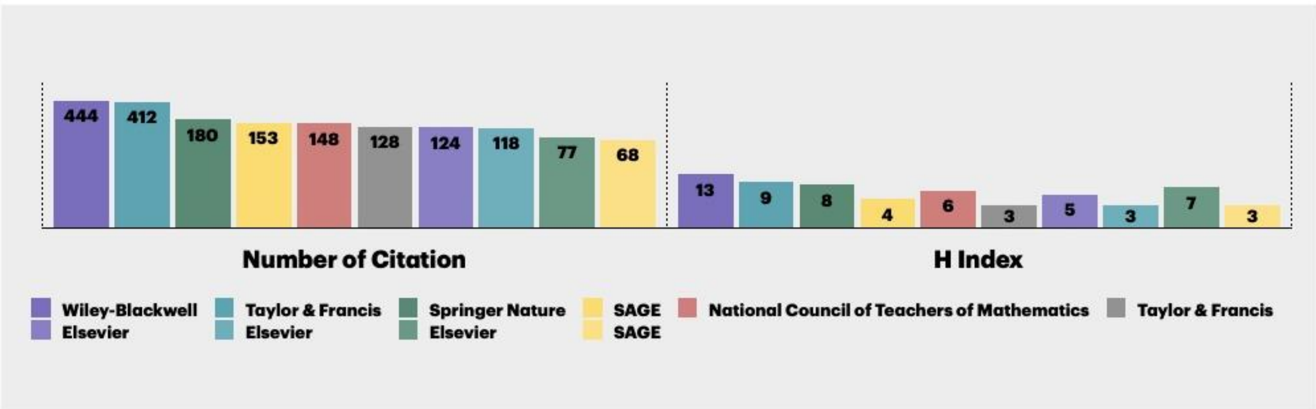


Figure (11): Citation statistics for the top 10 journals publishing on mathematical misconceptions

4.1.9. Impact of Authors

Ten authors in this field are connected to various universities and have published academic publications, as Table 8 illustrates. The contributors are connected to numerous academic institutions in several nations, such as South Africa, Belgium, Taiwan, Brunei Darussalam, Belgium, and the United States. The knowledge and insights produced by academic research are global in scope and overcome national borders because of this global representation.

An H-Index gives insight into an author's impact and influence beyond the number of documents. Notably, authors like Kelley Durkin and Bethany Rittle-Johnson, both affiliated with Vanderbilt University, boast H-Index scores of 6 and 5, respectively. These scores suggest a substantial scholarly impact, with their work being widely cited and recognized within their field.

Table (8): Top 10 most productive authors publishing in the field of mathematical misconceptions in terms of documents

Order	Authors	Institutions**	Number of documents	H-Index**
1	Kelley Durkin	Vanderbilt University	6	6
2	Judah P. Makonye	University of the Witwatersrand	6	3
3	Bethany Rittle-Johnson	Vanderbilt University	5	5
4	Lieven Verschaffel	University of Leuven	5	5
5	Der-Ching Yang	National Chiayi University	5	4
6	Wim Van Dooren	University of Leuven	4	4
7	Osman Birgin	Usak University	4	3
8	Masitah Shahrill	University Brunei Darussalam	4	3
9	Iwan A.J. Sianturi	Indiana University	4	3
10	Dirk De Bock	University of Leuven	3	3

** Number articles in the field of **mathematical misconceptions***
*** According to Scopus (http://www.scopus.com) dated Month xx, year*

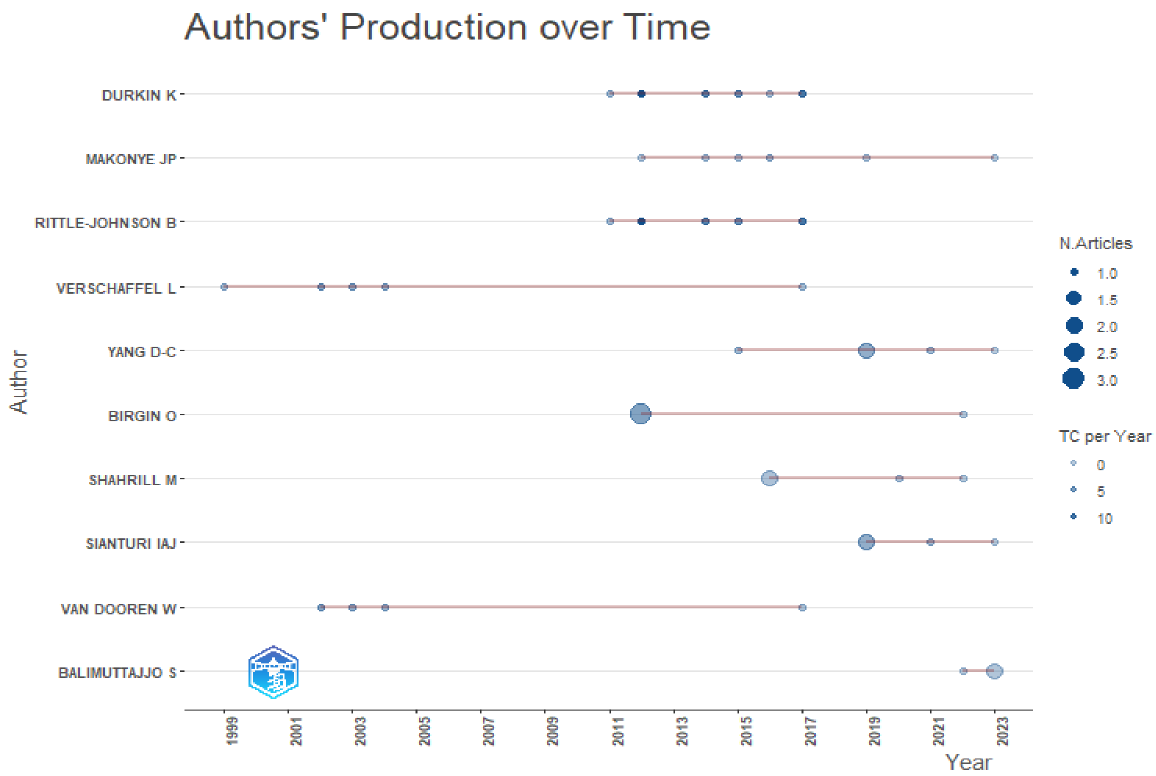


Figure (12): Top 10 most productive authors publishing in the field of mathematical misconceptions in terms of documents

Figure 12 shows the annual publications of the 10 most productive authors in the field of mathematical misconceptions. Across the ten authors, variations emerge in their publication records, with some individuals contributing more documents than others. For instance, authors like Kelley Durkin from Vanderbilt University and Judah P. Makonye from the University of the Witwatersrand have authored six documents each, indicating a significant scholarly output. On the other hand, authors such as Dirk De Bock from the University of Leuven have contributed three documents, reflecting a slightly lower level of publication activity.

Additionally, the figure shows that the authors in the top 10 have dedicated their attention to this issue during the past 10 years. Since 2017, Kelley Durkin, Bethany Rittle-Johnson, Lieven Verschaffel, and Wim Van Dooren have not produced any publications related to this topic.

The scholarly impact of 10 authors in their respective fields is displayed in Table 9 with a focus on the total number of citations that their work generated. The fact that each author is associated with a different university or research organization highlights the diversity of academic institutions that contribute to the larger intellectual landscape. There is a wide geographic distribution of expertise within the United States and beyond, indicating the global nature of knowledge production and dissemination in academia.

With an outstanding total of 1809 citations, Klaus Krippendorff is at the forefront of scholarly influence and has tremendous recognition and impact within the academic community. Jeremy M. Roschelle from the Institute for Research on Learning, Andrea A. Disessa from the University of California, and John P. Smith III from Michigan State University are just a few of the authors on the list. Each of them has more than 1000 citations, highlighting their significant contributions to the field of research.A fascinating aspect of the table is that several authors share affiliations with the same academic institutions. Gaea Leinhardt and Mary Kay Stein, for example, both attended the University of Pittsburgh, while Kelley Durkin and Bethany Rittle-Johnson attended Vanderbilt. This clustering of authors from common institutions suggests a collaborative research environment within these academic settings, potentially contributing to their collective scholarly impact.

Table (9): Top 10 most productive authors publishing in the field of mathematical misconceptions in terms of citations

Order	authors	Institutions**	Total Citations**
1	Klaus Krippendorff	University of Pennsylvania	1809
2	John P. Smith III	Michigan State University	1084
3	Jeremy M.Roschelle	Institute for Research on Learning	1080
4	Andrea A. Disessa	University of California	1080
5	Gaea Leinhardt	University of Pittsburgh	699
6	Mary Kay Stein	University of Pittsburgh	589
7	Orit Zaslavsky	Technion- Palestine	589
8	Kelley Durkin	Vanderbilt University	452
9	Bethany Rittle-Johnson	Vanderbilt University	441
10	Lieven Verschaffel	University of Leuven	263

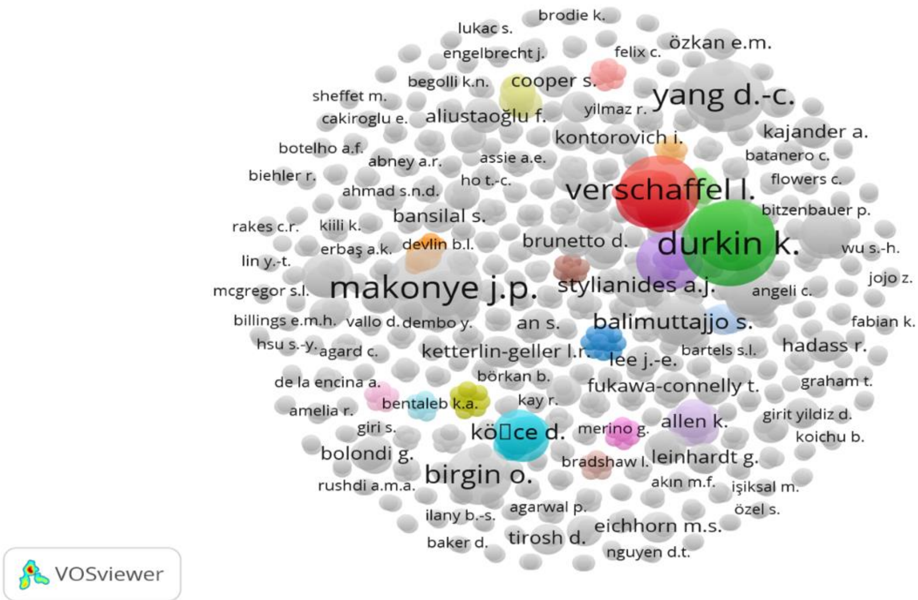


Figure (13): Collaborative network among authors in the field of mathematical misconceptions (VOS viewer)

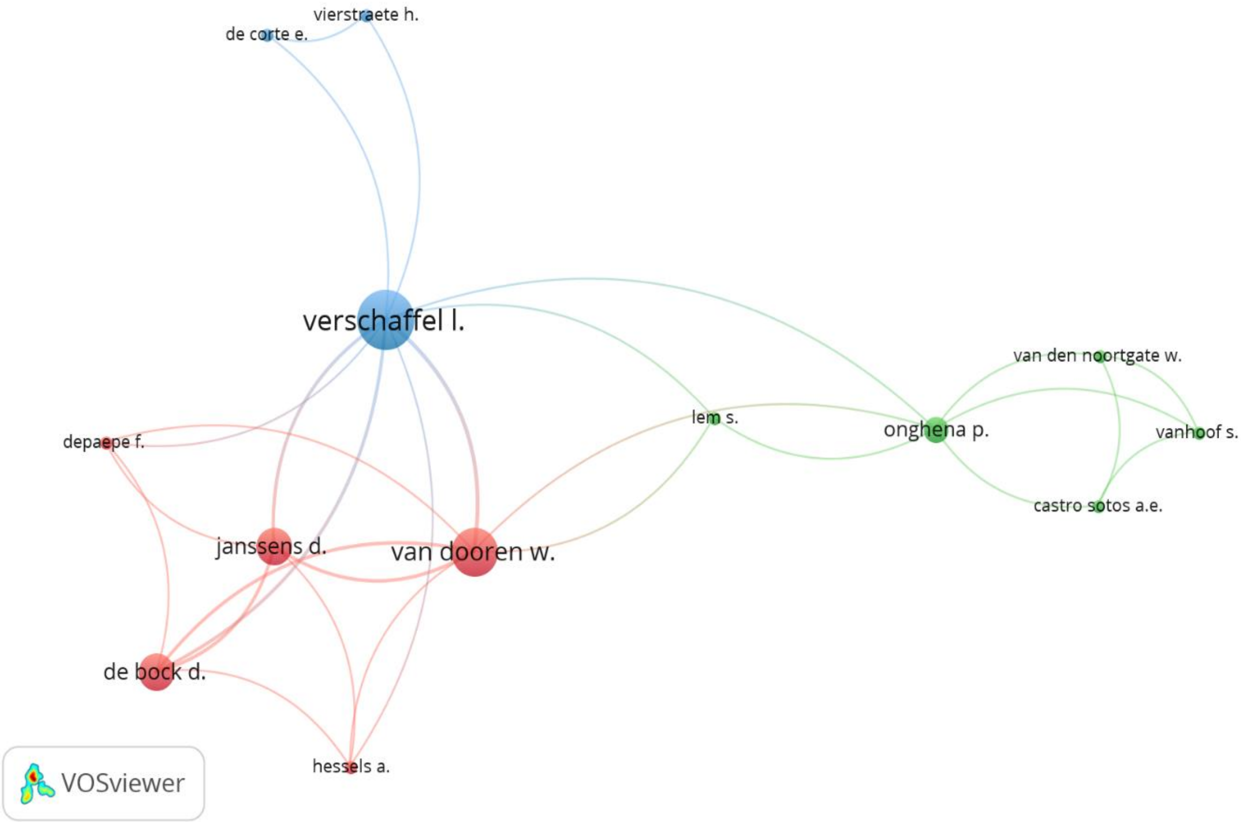


Figure (14): Collaborative network among authors in the field of mathematical misconceptions (VOS viewer)

For the analysis of collaboration between authors on mathematical misconceptions, all 1148 authors were included. The analysis revealed 318 different clusters (see Figure 13), in which only 3 groups were connected (see Figure 14), and a few researchers provided this connection. As shown in Figure 14, the relevant cooperative author groups are presented in different colors and represent the focus of mathematical misconceptions research. When we examine the co-authorship network visualization map in Figure 14, we understand that the cooperation between researchers working on mathematical misconceptions is low even though there are enough authors.

Also, the number of active researchers is weak in most groups. For example, out of the 318 groups, the most important groups are composed of a group of 13 authors, another with 11 authors, one with 10 authors, and yet another with 9 authors, 3 groups of 8 authors, 6 groups of 7 authors, 15 groups of 6 authors,17 groups of 5 authors, 46 groups of 4 authors, 91 groups of 3 authors, 123 groups of 2 authors, and 1 group of single authors.

The largest group is the red cluster composed of 5 authors, among which the authors with the most publications are Van DoorenW. (4 publications), Janssens d. and De Bock D. (3 publications). The second group is the green one, also composed of 5 authors. The most prominent in this group is Onghena P. (2 publications). The third-largest group is the one in blue. In this group of 3 authors, Verchaffel L. stands out with 5 publications.

4.2. The findings related to Content analysis and road map for future research of mathematics misconception.

4.2.1. Terms Analysis (topic trends)

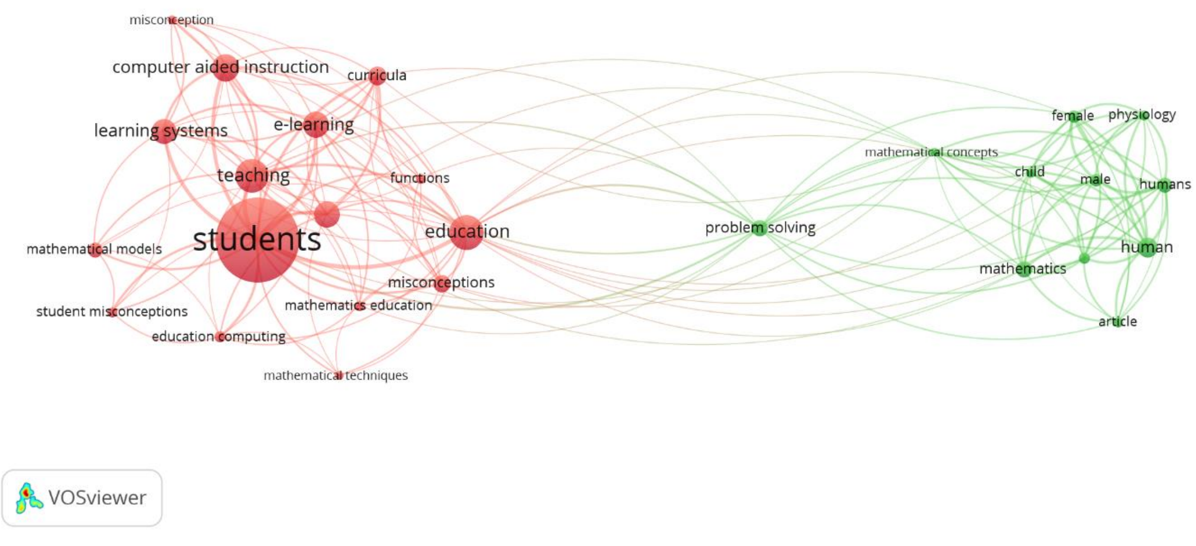


Figure (15): Co-occurrence network with index keywords in the field of mathematical misconceptions (VOS viewer)

A total of 823 keywords were used in 525 publications dealing with mathematical misconceptions. According to the co-occurrence analysis performed on the keywords, only 27 keywords appear in more than 5 publications. The co-occurrence analysis revealed that the keywords are grouped in 2 clusters or groups, as shown in Figure 15. The keywords in the clusters give information about related research topics in the area of interest.

As seen in Figure 15, the largest cluster is the red cluster with 16 keywords. Words like students, education, teaching, learning system, e-learning, and misconception, are highlighted. The green cluster is the second one with 11 keywords. These include humans, mathematics, and problem-solving are predominant.

Table 10 provides a breakdown of the occurrence of keywords across different intervals, allowing for an analysis of trends in academic literature over time.

Between 1947 and 1962, there were no occurrences of the specified keywords except for "Mathematics," which had 32 occurrences. This suggests that during this period, mathematical topics were a predominant focus in academic literature, while other keywords were relatively less prominent.

Similarly, from 1963 to 1978 and 1979 to 1994, there were no occurrences of the specified keywords except for "Mathematics," which maintained 32 and 38 occurrences, respectively. This indicates a continued emphasis on mathematical topics during these periods, with other keywords remaining relatively absent from scholarly discourse.

However, a notable shift occurs in the period from 1995 to 2010, where there is a significant increase in the occurrence of several keywords. "Students" emerged as a prominent focus with 125 occurrences, followed by "Mathematics" with 102 occurrences.

This period also sees moderate occurrences of keywords like "Education," "Teaching," "Computer Aided Instruction," "Female," "Learning System," and "Male," suggesting a broadening scope of research interests within the academic community.

The most substantial increase in keyword occurrences is observed in the period from 2011 to 2023. All keywords experience a significant increase in occurrences compared to previous periods, indicating a heightened interest in diverse topics related to education, technology, gender, and learning systems. "Students" remains the most prevalent keyword with 499 occurrences, followed by "Education," "Teaching," and "Male" with over 100 occurrences each.

Table 10. Word frequency over time in the field of mathematical misconceptions

Period	**Keywords occurrence**									
	Students	Education	Teaching	Computer Aided Instruction	Mathematics	E-Learning	Engineering Education	Female	Learning system	Male
1947 – 1962	0	0	0	0	32	0	0	0	0	0
1963-1978	0	0	0	0	32	0	0	0	0	0
1979-1994	0	0	0	0	38	0	0	0	0	0
1995-2010	125	69	50	59	102	5	18	50	51	50
2011-2023	499	232	197	128	155	102	122	132	133	132

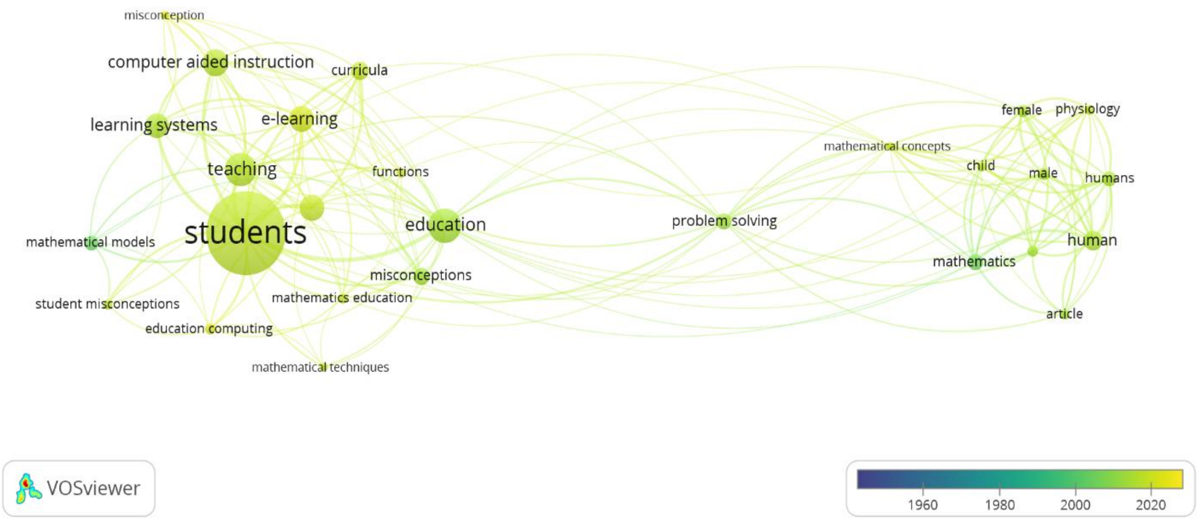


Figure (16): Overlay map showing the Co-occurrence network with index keywords in the field of mathematical misconceptions (VOS viewer)

The overlay map that emerged from the keyword co-occurrence analysis reveals the trend over the years, i.e., from 1947 to 2023, and gives an idea about the research topic's tendency. The overlay visualization map related to our analysis is shown in Figure 16. Analysing Figure 16, we notice that the search topics have almost not changed between 2000 and 2023 since there is a slight color variation. There was a change in research topics between 1947 and 1995, e.g., keywords like mathematics and mathematical models were more fully attended to than other keywords during the selected period.

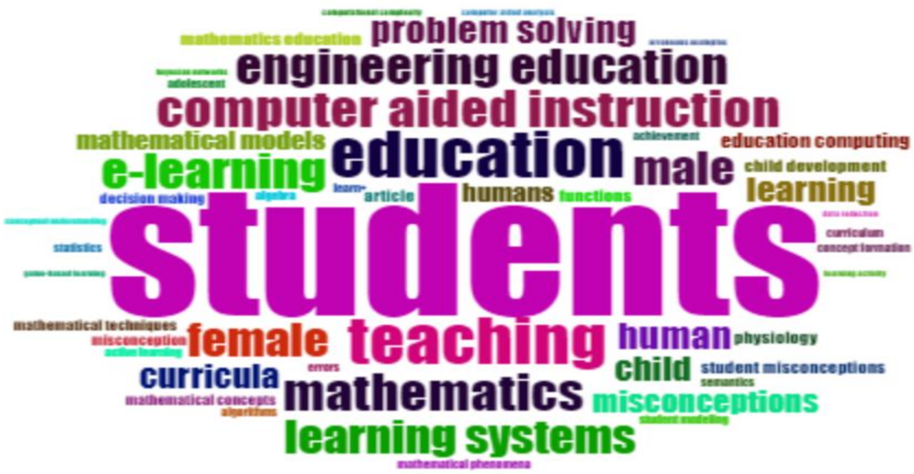


Figure (17): Words cloud (based on keywords) in the field of mathematical misconceptions

When considering the words cloud in Figure 17 created by Biblioshiny software based on keywords of the published publications on mathematical misconceptions, it can be seen that the main fields with the presence of mathematical misconceptions research are Students: 64 (14%), and Education and Teaching: 23 (5%). Besides these, we can see that the topics and subjects of research are very diverse.

4.2.2. The shift of Foci in the topic of interest

Table 11 presents a comprehensive overview of the frequency of various terms in academic research in the field of mathematical misconceptions over different intervals. It allows for a deeper analysis of trends and focuses within specific domains of study.

For instance, terms like "Mathematical Models" and "Problem Solving" show consistent but moderate frequencies across the years, suggesting enduring relevance in academic discourse. Terms such as "Misconceptions" showed a significant rise in occurrence from 2006 to 2016, indicating an increasing interest or concern in the research community.

Table 11. Mathematical misconceptions trend topics

Item	Freq	Year_q1	Year_med	Year_q3
students	64	2008	2016	2021
education	23	2003	2014	2017
teaching	23	2008	2015	2020
mathematics	18	1998	2013	2019
computer aided instruction	18	2003	2018	2023
engineering education	17	2013	2017	2021
e-learning	17	2014	2019	2022
learning systems	16	2004	2014	2021
female	16	2010	2016	2018
male	16	2010	2016	2018
problem solving	13	2003	2011	2017
learning	12	2007	2014	2017
misconceptions	11	2006	2010	2016
mathematical models	9	2000	2003	2006
humans	9	2009	2015	2017
education computing	7	2016	2021	2022
child development	6	2014	2015	2018
functions	6	2017	2018	2020
mathematical techniques	5	2010	2017	2019
misconception	5	2022	2023	2023

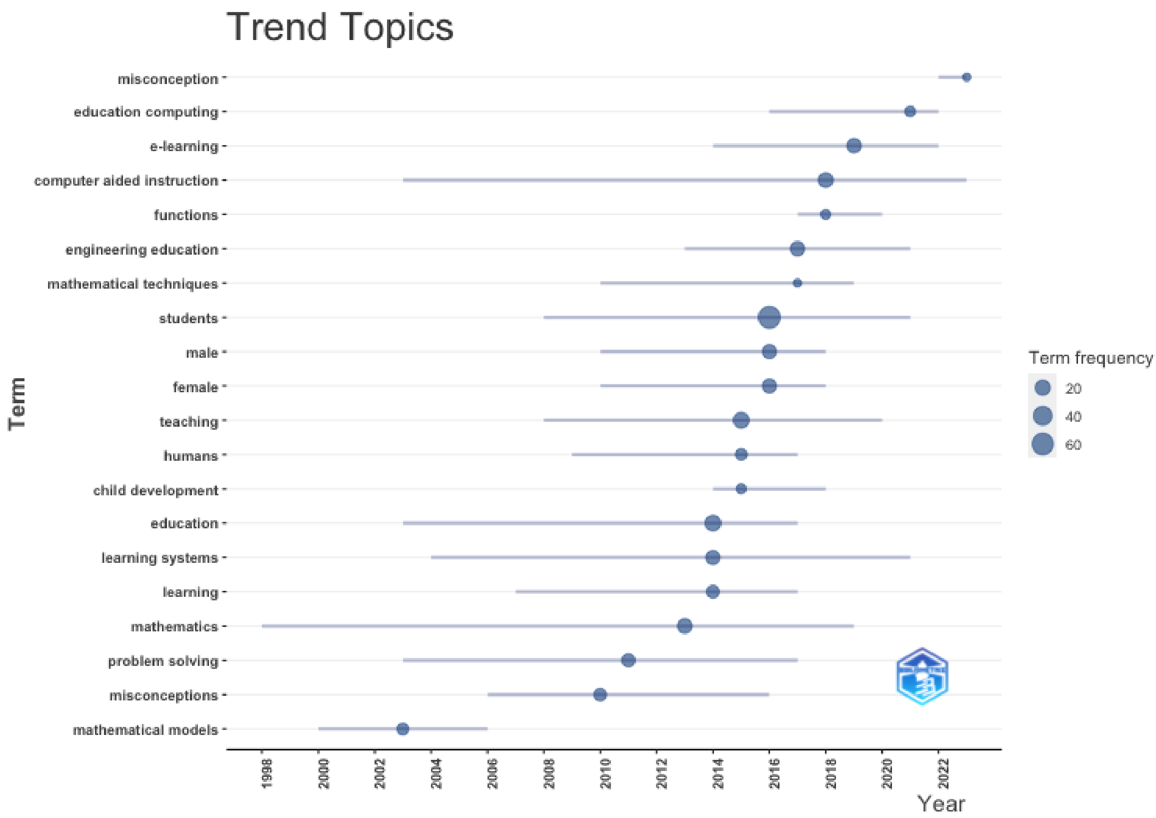


Figure (18): Topics trending in math misconceptions

Figure 18 shows how concentration and attention of trending topics vary over time. As an example, the term "Education" is used a lot, peaking around 2014, suggesting ongoing scholarly interest in educational subjects. Comparably, the terms "Teaching" and "Learning Systems" have become more common throughout time, suggesting a greater emphasis on teaching techniques.

Figure 18 also shows periods when certain topics have experienced heightened interest or research activity. For example, terms like "education computing" and "E-Learning" demonstrate significant increases in frequency in more recent years, reflecting the impact of technological advancements and changing educational paradigms.

4.2.3. The most common mathematics misconception

Based on the number of citations monitored in Table 5 through bibliometric analysis, Figure 19 shows the common mathematical misconceptions identified through content analysis across the first ten scientific papers (Krippendorff, 2004; Smith, 1993; Leinhardt et al., 1990; Tirosh, 2000; Sfard, 2001; Durkin & Rittle-Johnson, 2012; Wang, 2014; Newton, 2008; Leinhardt & Steele, 2005; Bennell et al., 2001).

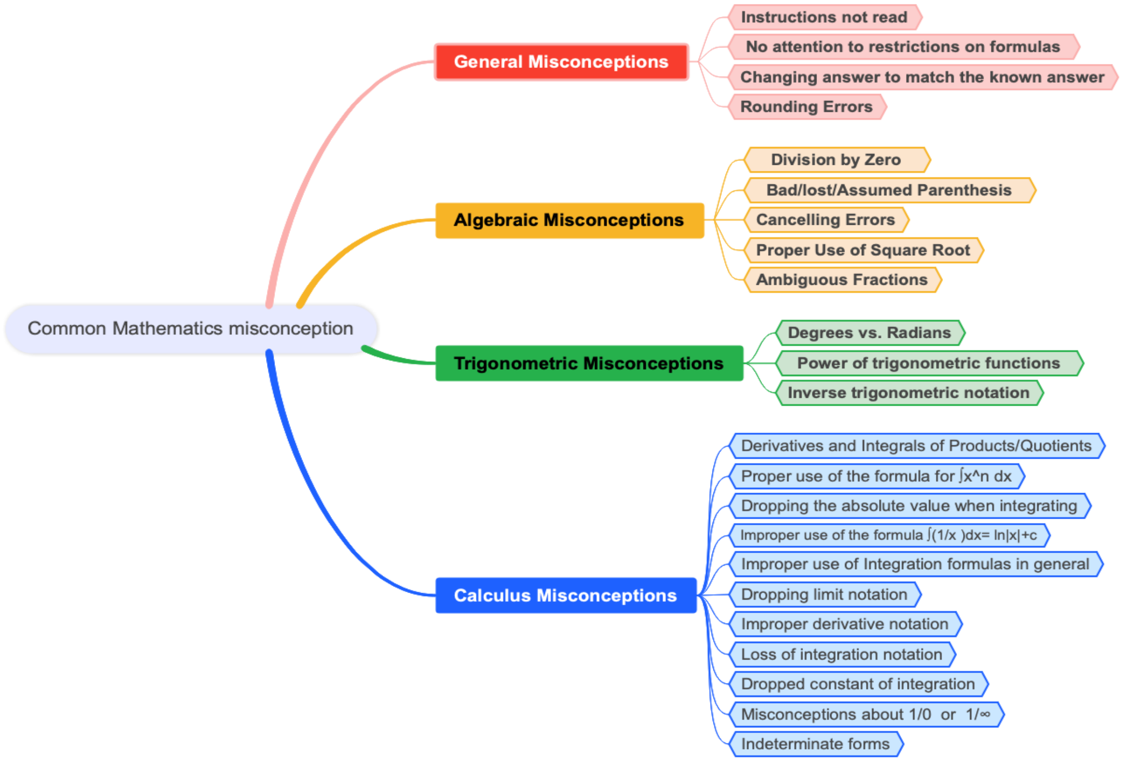


Figure (19): Common mathematical misconceptions according to error type

According to Figure 19, the most common mathematics misconceptions cover four areas. The number of general errors in the first field reached (4). The number of conceptual errors related to algebra reached (5). Trigonometric conceptual errors numbered (3), while calculus conceptual errors numbered (11).

4.2.3.1. General Misconceptions

Figure 20 shows the common mathematical misconceptions related to General Misconception

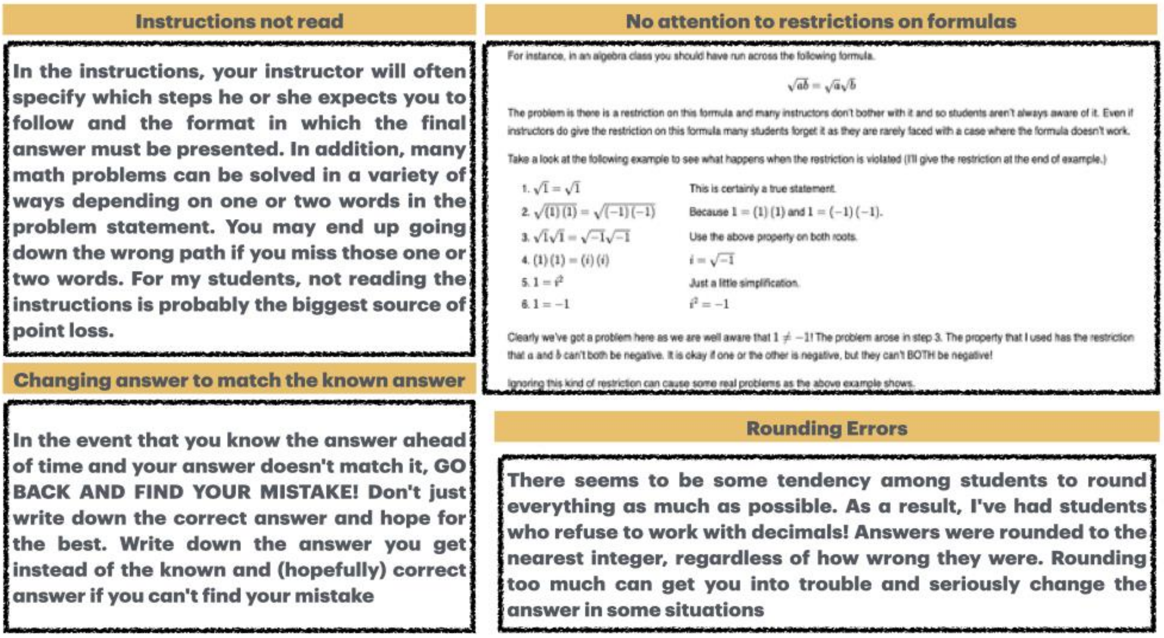


Figure (20): General mathematics misconceptions

4.2.3.2. Algebraic Misconceptions

Figure 21 shows the common mathematical misconceptions related to Algebraic Misconceptions.

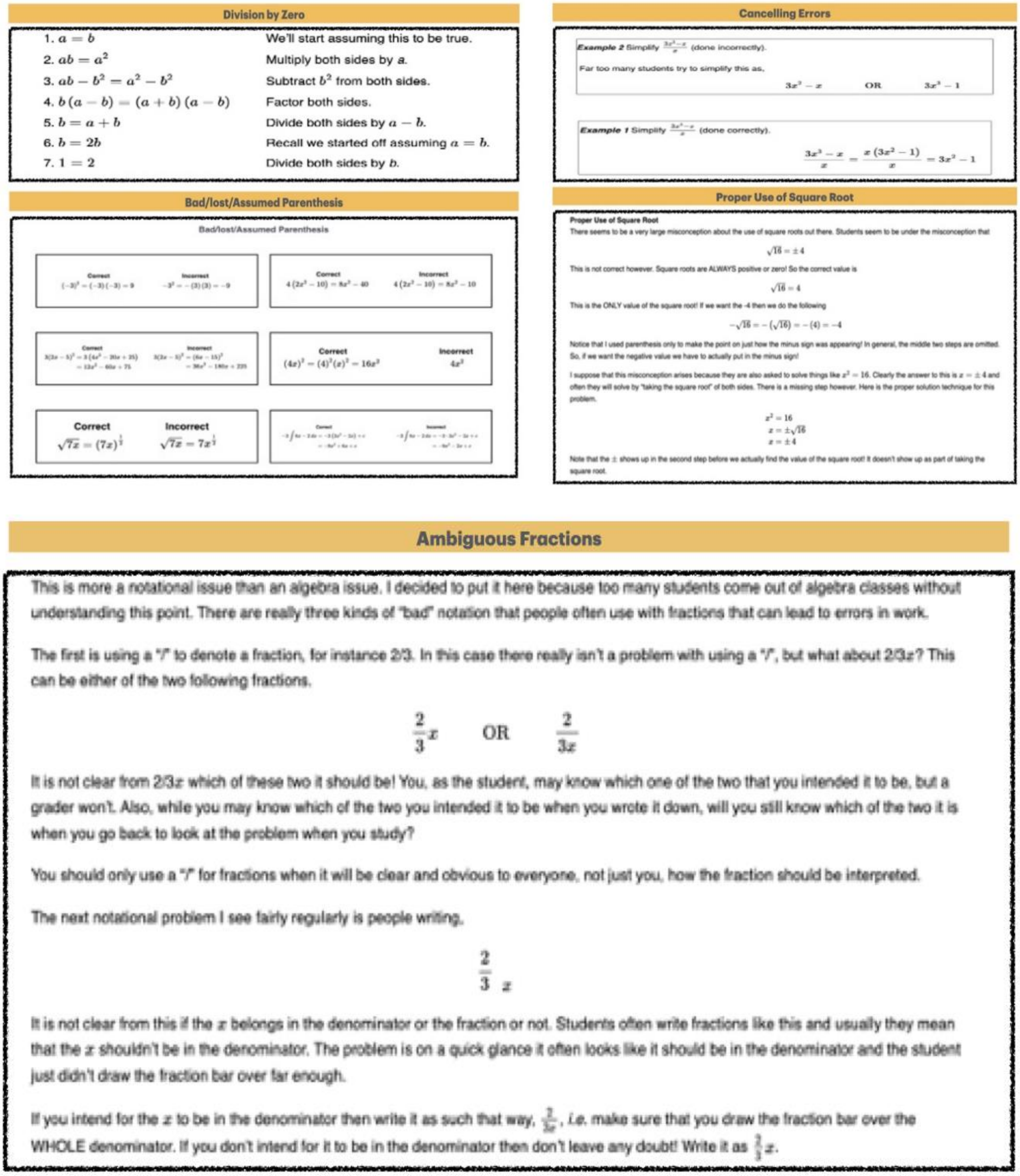


Figure (21): Algebraic mathematics misconceptions

4.2.3.3. Trigonometric Misconceptions

Figure 22 shows the common mathematical misconceptions related to Trigonometric Misconceptions.

Degrees vs. Radians

This is a fairly short section, but contains some errors that I see my calculus students continually making so I thought I'd include them here as a separate section.

Degrees vs. Radians

Most trig classes that I've seen taught tend to concentrate on doing things in degrees. I suppose that this is because it's easier for the students to visualize, but the reality is that almost all of calculus is done in radians and students too often come out of a trig class ill prepared to deal with all the radians in a calculus class.

You simply must get used to doing everything in radians in a calculus class. If you are asked to evaluate $\cos(x)$ at $x = 10$ we are asking you to use 10 radians not 10 degrees! The answers are very, very different! Consider the following.

$$\cos(10) = -0.839071529076 \qquad \text{in radians}$$
$$\cos(10) = 0.9848077753012 \qquad \text{in degrees}$$

You'll notice that they aren't even the same sign!

So, be careful and make sure that you always use radians when dealing with trig functions in a trig class. Make sure your calculator is set to calculations in radians.

$\cos(x)$ is NOT multiplication

I see students attempting both of the following on a continual basis

$$\cos(x + y) \neq \cos(x) + \cos(y)$$
$$\cos(3x) \neq 3\cos(x)$$

These just simply aren't true. The only reason that I can think of for these mistakes is that students must be thinking of $\cos(x)$ as a multiplication of something called $\cos$ and x. This couldn't be farther from the truth! Cosine is a function and the $\cos$ is used to denote that we are dealing with the cosine function!

If you're not sure you believe that those aren't true just pick a couple of values for x and y and plug into the first example.

$$\cos(\pi + 2\pi) \neq \cos(\pi) + \cos(2\pi)$$
$$\cos(3\pi) \neq -1 + 1$$
$$-1 \neq 0$$

So, it's clear that the first isn't true and we could do a similar test for the second example.

$$\cos(3\pi) \neq 3\cos(\pi)$$
$$-1 \neq 3(-1)$$
$$-1 \neq -3$$

I suppose that the problem is that occasionally there are values for these that are true. For instance, you could use $x = \frac{\pi}{2}$ in the second example and both sides would be zero so it would work for that value of x. In general, however, for the vast majority of values out there in the world these simply aren't true!

On a more general note. I picked on cosine for this example, but I could have used any of the six trig functions, so be careful!

Power of trigonometric functions

Remember that if n is a positive integer then

$$\sin^n x = (\sin x)^n$$

The same holds for all the other trig functions as well of course. This is just a notational idiosyncrasy that you've got to get used to. Also remember to keep the following straight.

$$\tan^2 x \qquad \text{vs.} \qquad \tan x^2$$

In the first case we taking the tangent then squaring result and in the second we are squaring the x then taking the tangent.

The $\tan x^2$ is actually not the best notation for this type of problem, but I see people (both students and instructors) using it all the time. We really should probably use $\tan(x^2)$ to make things clear.

Inverse trigonometric notation

The notation for inverse trig functions is not the best. You need to remember, that despite what I just got done talking about above,

$$\cos^{-1} x \neq \frac{1}{\cos x}$$

This is why I said that n was a positive integer in the previous discussion. I wanted to avoid this notational problem. The -1 in $\cos^{-1} x$ is NOT an exponent, it is there to denote the fact that we are dealing with an inverse trig function.

There is another notation for inverse trig functions that avoids this problem, but it is not always used.

$$\cos^{-1} x = \arccos x$$

Figure (22): Trigonometric mathematics misconceptions

4.2.3.4. Calculus Misconceptions

Figure 23 shows the common mathematical misconceptions related to Calculus Misconceptions.

Proper use of the formula for $\int x^n dx$

Many students forget that there is a restriction on this integration formula, so for the record here is the formula along with the restriction.

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c, \qquad \text{provided } n \neq -1$$

That restriction is incredibly important because if we allowed $n = -1$ we would get division by zero in the formula! Here is what I see far too many students do when faced with this integral.

$$\int x^{-1} dx = \frac{x^0}{0} + c = x^0 + c = 1 + c$$

THIS ISN'T TRUE!!!!!! There are all sorts of problems with this. First there's the improper use of the formula, then there is the division by zero problem! This should NEVER be done this way.

Recall that the correct integral of x^{-1} is,

$$\int x^{-1} dx = \int \frac{1}{x} dx = \ln|x| + c$$

Derivatives and Integrals of Products/ Quotients

Recall that while

$$(f \pm g)'(x) = f'(x) \pm g'(x) \qquad \int f(x) \pm g(x)\, dx = \int f(x)\, dx \pm \int g(x)\, dx$$

are true, the same thing can't be done for products and quotients. In other words,

$$(fg)'(x) \neq f'(x)\, g'(x) \qquad \int f(x)\, g(x)\, dx \neq \left(\int f(x)\, dx\right)\left(\int g(x)\, dx\right)$$

$$\left(\frac{f}{g}\right)'(x) \neq \frac{f'(x)}{g'(x)} \qquad \int \frac{f(x)}{g(x)}\, dx \neq \frac{\int f(x)\, dx}{\int g(x)\, dx}$$

If you need convincing of this consider the example of $f(x) = x^4$ and $g(x) = x^{10}$.

$$(fg)'(x) \neq f'(x)\, g'(x)$$
$$(x^4 x^{10})' \neq (x^4)'(x^{10})'$$
$$(x^{14})' \neq (4x^3)(10x^9)$$
$$14x^{13} \neq 40x^{12}$$

I only did the case of the derivative of a product, but clearly the two aren't equal! I'll leave it to you to check the remaining three cases if you'd like to.

Remember that in the case of derivatives we've got the product and quotient rule. In the case of integrals there are no such rules and when faced with an integral of a product or quotient they will have to be dealt with on a case by case basis.

Improper use of the formula $\int \frac{1}{x} dx = \ln(|x|) + c$

Gotten the impression yet that there are more than a few mistakes made by students when integrating $\frac{1}{x}$? I hope so, because many students lose huge amounts of points on these mistakes. This is the last one that I'll be covering however.

In this case, students seem to make the mistake of assuming that if $\frac{1}{x}$ integrates to $\ln|x|$ then so must one over anything! The following table gives some examples of incorrect uses of this formula.

Integral	Incorrect Answer	Correct Answer
$\int \frac{1}{x^2+1} dx$	$\ln(x^2+1) + c$	$\tan^{-1}(x) + c$
$\int \frac{1}{x^2} dx$	$\ln(x^2) + c$	$-x^{-1} + c = -\frac{1}{x} + c$
$\int \frac{1}{\cos x} dx$	$\ln|\cos x| + c$	$\ln|\sec x + \tan x| + c$

So, be careful when attempting to use this formula. This formula can only be used when the integral is of the form $\int \frac{1}{x} dx$. Often, an integral can be written in this form with an appropriate u-substitution (the two integrals from previous example for instance), but if it can't be then the integral will NOT use this formula so don't try to.

Dropping the absolute value when integrating

$$\int \frac{1}{x} dx$$

Recall that in the formula

$$\int \frac{1}{x} dx = \ln|x| + c$$

the absolute value bars on the argument are required! It is certainly true that on occasion they can be dropped after the integration is done, but they are required in most cases. For instance, contrast the two integrals,

$$\int \frac{2x}{x^2+10} dx = \ln|x^2+10| + C = \ln(x^2+10) + c$$
$$\int \frac{2x}{x^2-10} dx = \ln|x^2-10| + c$$

In the first case the x^2 is positive and adding 10 on will not change that fact so since $x^2 + 10 > 0$ we can drop the absolute value bars. In the second case however, since we don't know what the value of x is, there is no way to know the sign of $x^2 - 10$ and so the absolute value bars are required.

Improper use of Integration formulas in general

This one is really the same issue as the previous one, but so many students have trouble with logarithms that I wanted to treat that example separately to make the point.

So, as with the previous issue students tend to try and use "simple" formulas that they know to be true on integrals that, on the surface, kind of look the same. So, for instance we've got the following two formulas.

$$\int \sqrt{u}\,du = \frac{2}{3}u^{\frac{3}{2}} + C$$
$$\int u^2\,du = \frac{1}{3}u^3 + C$$

The mistake here is to assume that if these are true then the following must also be true.

$$\int \sqrt{\text{anything}}\,du = \frac{2}{3}(\text{anything})^{\frac{3}{2}} + C$$
$$\int (\text{anything})^2\,du = \frac{1}{3}(\text{anything})^3 + C$$

This just isn't true! The first set of formulas work because it is the square root of a single variable or a single variable squared. If there is anything other than a single u under the square root or being squared then those formulas are worthless. On occasion these will hold for things other than a single u, but in general they won't hold so be careful!

Here's another table with a couple of examples of these formulas not being used correctly.

Integral	Incorrect Answer	Correct Answer
$\int \sqrt{x^2+1}\,dx$	$\frac{2}{3}(x^2+1)^{\frac{3}{2}} + C$	$\frac{1}{2}\left(x\sqrt{x^2+1} + \ln\lvert x + \sqrt{x^2+1}\rvert\right) + C$
$\int \cos^2 x\,dx$	$\frac{1}{3}\cos^3 x + C$	$\frac{x}{2} + \frac{1}{4}\sin(2x) + C$

If you aren't convinced that the incorrect answers really aren't correct then remember that you can always check you answers to indefinite integrals by differentiating the answer. If you did everything correctly you should get the function you originally integrated, although in each case it will take some simplification to get the answers to be the same.

Also, if you don't see how to get the correct answer for these they typically show up in a Calculus II class. The second however, you could do with only Calculus I under your belt if you can remember an appropriate trig formula.

Improper derivative notation

When asked to differentiate $f(x) = x(x^3 - 2)$ I will get the following for an answer on occasion.

$$\begin{aligned} f(x) &= x(x^3-2) \\ &= x^4 - 2x \\ &= 4x^3 - 2 \end{aligned}$$

This is again a situation where you may know what you're intending to say here, but anyone else who reads this will come away with the idea that $x^4 - 2x = 4x^3 - 2$ and that is clearly NOT what you are trying to say. However, it IS what you are saying when you write it this way.

The proper notation is

$$\begin{aligned} f(x) &= x(x^3-2) \\ &= x^4 - 2x \\ f'(x) &= 4x^3 - 2 \end{aligned}$$

Dropping limit notation

The remainder of the errors in this document consists mostly of notational errors that students tend to make.

I'll start with limits. Students tend to get lazy and start dropping limit notation after the first step. For example, an incorrectly worked problem is

$$\lim_{x\to 3}\frac{x^2-9}{x-3} = \frac{(x-3)(x+3)}{x-3} = x+3 = 6$$

There are several things wrong with this. First, when you drop the limit symbol you are saying that you've in fact taken the limit. So, in the first equality,

$$\lim_{x\to 3}\frac{x^2-9}{x-3} = \frac{(x-3)(x+3)}{x-3}$$

you are saying that the value of the limit is

$$\frac{(x-3)(x+3)}{x-3}$$

and this is clearly not the case. Also, in the final equality,

$$x + 3 = 6$$

you are making the claim that each side is the same, but this is only true provided $x = 3$ and what you really are trying to say is

$$\lim_{x\to 3} x + 3 = 6$$

You may know what you mean, but someone else will have a very hard time deciphering your work. Also, your instructor will not know what you mean by this and won't know if you understand that the limit symbols are required in every step until you actually take the limit. If you are one of my students, I won't even try to read your mind and I will assume that you didn't understand and take points off accordingly.

So, while you may feel that it is silly and unnecessary to write limits down at every step it is proper notation and in my class I expect you to use proper notation. The correct way to work this limit is.

$$\lim_{x\to 3}\frac{x^2-9}{x-3} = \lim_{x\to 3}\frac{(x-3)(x+3)}{x-3} = \lim_{x\to 3} x+3 = 6$$

The limit is required at every step until you actually take the limit, at which point the limit must be dropped as I have done above.

Loss of integration notation

There are many dropped notation errors that occur with integrals. Let's start with this example.

$$\int x(3x-2)\,dx = 3x^2 - 2x = x^3 - x^2 + c$$

As with the derivative example above, both of these equalities are incorrect. The minute you drop the integral sign you are saying that you've done the integral! So, this means that the first equality is saying that the value of the integral is $3x^2 - 2x$, when in reality all you're doing is simplifying the function. Likewise, the last equality says that the two functions, $3x^2 - 2x$ and $x^3 - x^2 + c$ are equal, when they are not! Here is the correct way to work this problem.

$$\int x(3x-2)\,dx = \int 3x^2 - 2x\,dx = x^3 - x^2 + c$$

Another big problem in dropped notation is students dropping the dx at the end of the integrals. For instance,

$$\int 3x^2 - 2x$$

The problem with this is that the dx tells us where the integral stops! So, this can mean a couple of different things.

$$\int 3x^2 - 2x\,dx = x^3 - x^2 + c \qquad \text{OR}$$
$$\int 3x^2 dx - 2x = x^3 - 2x + c$$

Without the dx a reader is left to try and intuit where exactly the integral ends! The best way to think of this is that parenthesis always come in pairs "(" and ")". You don't open a set of parenthesis without closing it. Likewise, $\int$ is always paired up with a dx. You can always think of $\int$ as the opening parenthesis and the dx as the closing parenthesis.

Another dropped notation error that I see on a regular basis is with definite integrals. Students tend to drop the limits of integration after the first step and do the rest of the problem with *implied* limits of integration as follows.

$$\int_1^2 x(3x-2)\,dx = \int 3x^2 - 2x\,dx = x^3 - x^2 = 8 - 4 - (1-1) = 4$$

Again, the first equality here just doesn't make sense! The answer to a definite integral is a number, while the answer to an indefinite integral is a function. When written as above you are saying the answer to the definite integral and the answer to the indefinite integral are the same when they clearly aren't!

Likewise, the second to last equality just doesn't make sense. Here you are saying that the function, $x^3 - x^2$ is equal to $8 - 4 - (1-1) = 4$ and again, this just isn't true! Here is the correct way to work this problem.

$$\int_1^2 x(3x-2)\,dx = \int_1^2 3x^2 - 2x\,dx = \left(x^3 - x^2\right)\Big|_1^2 = 8 - 4 - (1-1) = 4$$

Dropped constant of integration

One of the biggest errors students make in integration is omitting the constant of integration (the +c+c part). Students make two errors here. Some students don't include it at all, while others drop it from intermediate steps and just add it at the end

Misconceptions about $\frac{1}{0}$ or $\frac{1}{\infty}$

This is not so much about an actual error that students make, but instead a misconception that can, on occasion, lead to errors. This is also a misconception that is often encouraged by laziness on the part of the instructor.

So, just what is this misconception? Often, we will write $\frac{1}{\infty} = 0$ and $\frac{1}{0} = \infty$. The problem is that neither of these are technically correct and in fact the second, depending on the situation, can actually be $\frac{1}{0} = -\infty$. All three of these are really limits and we just short hand them. What we really should write is

$$\lim_{x \to \infty} \frac{1}{x} = 0$$
$$\lim_{x \to 0^+} \frac{1}{x} = \infty$$
$$\lim_{x \to 0^-} \frac{1}{x} = -\infty$$

In the first case 1 over something increasingly large is increasingly small and so **in the limit** we get zero. In the last two cases note that we've got to use one-sided limits as $\lim_{x \to 0} \frac{1}{x}$ doesn't even exist! In these two cases, 1 over something increasingly small is increasingly large and will have the sign of the denominator and so in the limit it goes to either ∞ or $-\infty$.

Indeterminate forms

This is actually a generalization of the previous topic. The two operations above, $\infty - \infty$ and $\frac{\infty}{\infty}$ are called *indeterminate forms* because there is no one single value for them. Depending on the situation they have a very wide range of possible answers.

There are many more indeterminate forms that you need to look out for. As with the previous discussion there is no way to determine their value without taking the situation into consideration. Here are a few of the more common indeterminate forms.

$$\infty - \infty \qquad \frac{\infty}{\infty} \qquad \frac{0}{0} \qquad 0 \cdot \infty$$
$$0^0 \qquad 1^\infty \qquad \infty^0$$

Let's just take a brief look at 0^0 to see the potential problems. Here we really have two separate rules that are at odds with each other. Typically we have $0^n = 0$ (provided n is positive) and $a^0 = 1$. Each of these rules implies that we could get different answers. Depending on the situation we could get either 0 or 1 as an answer here. In fact, it's also possible to get something totally different from 0 or 1 as an answer here as well.

All the others listed here have similar problems. So, when dealing with indeterminate forms you need to be careful and not jump to conclusions about the value.

Figure (23): Calculus mathematics misconceptions

CHAPTER FIVE

Discussion and Recommendations

This chapter includes discussion of the study results and recommendations for future improvements. Research questions and drawn conclusions are discussed separately.

5.1. Discussion and Conclusions related to Performance Analysis and benchmarking of mathematics misconception.

5.1.1. Overview of Mathematical Misconceptions.

The substantial number of documents (525) underscores the breadth of research covered in the study. Notably, the breakdown of document types reveals a predominant focus on articles (447), demonstrating the scholarly prioritization of thorough research and analysis over conference presentations.

The distribution of single-authored documents (133) alongside the prevalence of co-authors per document (2.45) underscores the collaborative nature of scholarly endeavors, suggesting a trend towards interdisciplinary collaboration and knowledge exchange.

5.1.2. Publication Output and Growth Trend

There was significant academic growth and influence between the years 1979 to 2010, This trend aligns with broader advancements in technology, communication, and interdisciplinary collaboration during this period, which likely facilitated increased research dissemination and citation rates.

In the interval spanning from 2011 to 2023, the total citation count of 104 stands out as notably lower compared to previous periods. This disparity can be attributed to the relative recency of publications within this timeframe, as scholarly works often require time to gain visibility and attract citations from peers. Consequently, the lower citation count in this period should not be misconstrued as indicative of diminished scholarly impact or quality.

5.1.3. & 5.1.4 An Overview of Country Production Over Time and Nations' Contributions

In the 1990s, NCTM (National Council of Teachers of Mathematics) revised and updated its Curriculum and Evaluation Standards which explains the significant increase in the article's output that was observed from 1995 to 2010, particularly from the USA, which contributed the most to scholarly literature during this time frame, From 2011 to 2023, article production increased dramatically across all countries, with the USA, Indonesia, and Turkey bringing the most notable contributions. This growth was probably caused by technological advancements, globalization, and research infrastructure, which made publishing platforms and international cooperation more accessible (Neidorf et al., 2020).

5.1.5. Contribution by Affiliations

With nine publications, the University of the Witwatersrand in South Africa is the leading contributor, highlighting its significance in the academic community. Due to the diversity of affiliations, the field of mathematical misconceptions is experiencing wide interest, not only because of the global nature of academic collaboration but also due to the wide interest in the field. Scholars worldwide have participated in this research topic, indicating its relevance and significance in numerous educational contexts.

Global participation in mathematics education emphasizes the value of collaborative learning environments and cross-cultural views. Additionally, it indicates that researchers from various areas are dedicated to expanding our knowledge of the underlying reasons for misconceptions and the most efficient ways to correct them in the classroom. (Kadarisma, 2020).

5.1.6. Contribution by document

The articles entitled "Misconceptions Reconceived: An Analysis of Knowledge in Transition" and "Reliability in Content Analysis: Some Common Misconceptions and Recommendations" have received the most citations. The two top papers have much different citation counts than the other research, which points to a concerning pattern in scholarly referencing. This suggests academics frequently use well-known works as sources of reference while ignoring significant contributions made by other authors.

5.1.7. Journal impact according to documents

Journals like Educational Studies in Mathematics, International Journal of Mathematical Education in Science and Technology, and Journal of Mathematical Behavior consistently discuss mathematical misconceptions. Some of these journals were listed among the most prestigious journals, such as the Journal of Mathematical Behavior and Educational Studies in Mathematics. (Nivens & Otten, 2017).

5.1.8. Journal impact according to citation

The 10 of the most referenced journals in the topic of mathematical misconceptions are located in the first quartile of their respective Scopus rankings. This relationship between high citation counts and first quartile status indicates a strong association between journal quality and citation impact. It suggests that journals in the first quartile get more citations, reflecting their reputation and power in the academic community.

5.1.9. Impact of Authors

The authors' H-index values and document counts varies considerably from one another. Some authors, like Bethany Rittle-Johnson from Vanderbilt University and Lieven Verschaffel from the University of Leuven, contributed fewer documents but a comparable H-index value. Other authors, like Kelley Durkin from Vanderbilt University and Judah P. Makonye from the University of Leuven, contributed more documents. ideally, an article's productivity and influence can be determined using the H-index and the quantity of documents (Julius et al., 2021).

The lack of cooperation among writers of mathematical misconceptions could be caused by a number of conditions. Researchers may choose independent research over group projects to optimize their academic performance, which may not result in immediate recognition of an individual. In addition, Coordination of collaborations can also be impacted by practical difficulties like meeting research objectives, communicating clearly, and getting across institutional obstacles.

5.2. Discussion, and conclusions about the content analysis and future plans for investigation in math misconceptions

5.2.1. Terms Analysis (topic trends)

The term "Mathematics" was continuously presented from 1947 until 2023, demonstrating its ongoing importance in the learning environment. This makes it evident that mathematics is an essential component of education and problem-solving skills. Furthermore, it's critical to acknowledge that misconceptions about mathematics continue to exist (Ay, 2017).

Since students play a crucial part in the educational process, it is consistent that the term "students" is used to refer to them most frequently. The researchers in this study supported the essential assumption that students' needs, goals, and development are at the center of education.

The growing use of terms as "e-learning" and "computer-assisted instruction" in the period after 1995 indicates that technology is playing a bigger role in education. Such a trend illustrates the necessity for educators to adjust to new technology in order to satisfy the demands of this evolution, which shows how educational methods are always changing (Djeki et al., 2022).

5.2.2. The chnge of Foci's area of interest

Over time, words that appear frequently like "education," "teaching," and "students" have maintained their significance. They are constantly present, yet they are also essential to education and learning. Fundamental to the educational system, the concepts of "education" and "teaching" are based on continuous debates and study in all academic and educational domains. Similarly, the prevalence of "students" indicates how crucial it is to concentrate learning efforts on learners.

Terms like "problem-solving," "mathematical models," and "math" have proliferated over time, highlighting the significance of mathematics education and the need to advance mathematical literacy and skill. Moreover, the terms "misconceptions" and "child development" are frequently used, which indicates that researchers are becoming more conscious of the cognitive and developmental components of learning.

5.2.3. The most common misconception in mathematics

According to McDonald (2010), misconceptions are common in many areas of mathematics, including computation, algebra, geometry, trigonometry, linear equations, quadratic equations, similar triangle relations, and functions. Based on our qualitative content analysis, we identified four major categories of misconceptions: general errors, algebraic misconceptions, conceptual errors in trigonometry, and conceptual errors in calculus.

The consistency of misconceptions highlights the need for pedagogical approaches and instructional interventions that address them effectively. Teachers can create ways for addressing deeper conceptual understanding, identifying and correcting common errors, and enhancing students' mathematical reasoning skills as they become aware of the persistent nature of these misconceptions. (Kshetree et al., 2021).

5.3. Recommendations

In the light of the above-mentioned conclusions, this study recommends the following:

- Researchers should extend their investigations of bibliometric analysis to other fields of education.
- Based on the findings of this bibliometric analysis, which focused exclusively on English-language articles related to mathematics misconceptions, future researchers should consider conducting a similar bibliometric analysis on Arabic-language articles related to mathematics misconceptions.
- To facilitate joint research efforts on mathematics misconceptions, countries should allocate funding and promote international collaboration.
- Curriculum planners are encouraged to participate in workshops designed to equip them with tools and knowledge to proactively address and prevent students from falling into common misconceptions in mathematics.
- Organizing teacher and educator workshops to address common misconceptions about mathematics that have been found through research.
- Offering extra classes or sessions for students to address Misconceptions about mathematics. Students would have the chance to talk about and clear up any misunderstandings they might have had during normal class periods during these sessions.
- Implementing formative assessment strategies in the classroom, teachers may help teachers keep an eye on their students' development and correct recurring misconceptions. Instructors can assist students in overcoming obstacles and mastering fundamental mathematical abilities by using frequent evaluations of their learning and correcting misconceptions as they occur.

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